THOMSON
NELSON

Additional Curriculum Support: Suggested Implementation Guide

The Blackline Masters listed below are found at the back of the corresponding Teacher's Resource Chapter booklets. These Blackline Masters supplement coverage of *The Ontario Curriculum, Grades 1-8: Mathematics, 2005*.

The chart below provides a suggested order for their implementation.

Chapter	Code	Title	Corresponding *Nelson Mathematics 7 Student Book Lesson*
2	2A	Adding and Subtracting Decimals Mentally	after Lesson 2.6
2	2B	Multiplying Decimals by Whole Numbers	before Lesson 2.7
2	2C	Dividing Decimals by Whole Numbers	before Lesson 2.8
2	2D	Solving Unit Rate Problems	after Lesson 2.8
2	2E	Dividing Whole Number by Decimal	after Lesson 2.8
3	3A	Circle Graph	after Lesson 3.2
3	3B	Central Tendency	after Lesson 3.6
3	3C	Analyzing Misleading Graphs	after Lesson 3.7
5	5A	Solving Area Problems	after Lesson 5.6
7	7A	Sorting Triangles and Quadrilaterals	before Lesson 7.5
7	7B	Relationships for Congruent Shapes	after Lesson 7.5
7	7C	Dilatations with Pattern Blocks	after Lesson 7.9
7	7D	Dilatations	after Lesson 7.9
7	7E	Constructing Angle Bisector	after Lesson 7.9
7	7F	Constructing Perpendicular Bisector	after Lesson 7.9
7	7G	Constructing Perpendicular Lines	after Lesson 7.9
7	7H	Constructing Parallel Lines	after Lesson 7.9
7	7I	Constructing Intersecting Lines	after Lesson 7.9
9	9A	Comparing Fractions and Decimals	before Lesson 9.1
9	9B	Dividing a Whole Number by a Fraction	after Lesson 9.5
9	9C	Adding and Subtracting Mentally	after Lesson 9.8
10	10A	Describing Right Prisms	after Lesson 10.1
11	11A	Surface Area of a Right Prism	after Lesson 11.1
11	11B	Volume of a Right Prism	after Lesson 11.2

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Chapter 2 Expectations and Answers

2A Adding and Subtracting Decimals Mentally (Exploration)

Expectations

- [demonstrate an understanding of addition and subtraction of fractions and integers, and] apply a variety of computational strategies to solve problems involving [whole numbers and] decimal numbers
- use a variety of mental strategies to solve problems involving the addition and subtraction of [fractions and] decimals
- solve multi-step problems arising from real-life contexts and involving [whole numbers and] decimals, using a variety of tools and strategies

Answers

A. 3.25 km, 1.75 km; 5 km; For example, 0.25 + 0.75 = 1, 3 + 1 = 4, 1 + 4 = 5

B. 5 + 2 + 1 = 8

C. 8 + 0.50 = 8.50, 8.50 + 0.35 = 8.85; 8.85 km

D. 0.65 km. For example, 9.50 – 8 = 1.5, 0.85 = 0.5 + 0.3 + 0.05, 1.5 – 0.5 = 1, 1 – 0.3 = 0.7, 0.7 – 0.05 = 0.65

E. For example, yes because it gives a visual representation of the trail.
For example, no because I could use just the numbers in the problem.

1. a) For example, James subtracted 2.50 from 9.50 to get 7. He added 1.75 and 3.25 to get 5. He subtracted their sum from 7 to get 2. He subtracted the whole number part of 1.35 from 2 to get 1. He subtacted the decimal part of 1.35 from 1 to get 0.65.

b) For example, 1 + 2 + 3 + 1 = 7, 0.25 + 0.75 = 1, 9.5 – 7 – 1 = 1.5, 1.5 – 0.5 = 1, 1 – 0.35 = 0.65; 0.65 km

c) For example, I prefer the strategy in steps A to D. I found it easier to solve the problem by adding the decimal parts for 1 whole first.

2. a) For example, Simon added the whole numbers. He thought of 0.8 as 0.3 and 0.5 and added the 0.3 to 12.7 to get 13. He added the remaining 0.5 to get 13.5 km.

b) For example, 6 + 2 = 8, 8 + 0.9 = 8.9, 8.9 + 0.1 = 9, 9 + 0.3 = 9.3; 9.3 km

3. a) For example, Jody subtracted 5 from 9.3 to get 4.3. Then she thought of 0.8 as 0.3 + 0.5 and subtracted 0.3 from 4.3 to get 4, and then subtracted the remaining 0.5 from 4 to get 3.5 km.

b) For example, 6.4 – 3 = 3.4. Since 0.7 = 0.4 + 0.3, subtract 0.4, and then 0.3. 3.4 – 0.4 = 3, 3 – 0.3 = 2.7; 2.7 km

4. a) For example, I want to run 10 km each week. On Monday, I ran 2.2 km, on Wednesday I ran 3.1 km, and on Friday I ran 0.9 km. How much farther do I have to run to meet the 10 km goal?

b) 3.1 + 0.9, 4 + 2.2, 10 – 6.2 = 3.8 km. I grouped the whole number and decimals that added up to a whole number. I added the other amount and then subtracted from 10.

2B Multiplying Decimals by Whole Numbers (Direct Instruction)

Expectations

- [demonstrate an understanding of addition and subtraction of fractions and integers, and] apply a variety of computational strategies to solve problems involving whole numbers and decimal numbers
- solve problems involving the multiplication [and division] of decimal numbers to thousandths by one-digit whole numbers, using a variety of tools and strategies
- solve multi-step problems arising from real-life contexts and involving whole numbers and decimals, using a variety of tools and strategies
- use estimation when solving problems involving operations with whole numbers, decimals[, and percents,] to help judge the reasonableness of a solution

Answers

1. For example, Rana modelled the four groups. The thousandths totalled 12 so she regrouped 10 thousandths as 1 hundredth making 17 hundredths. She regrouped 10 hundredths as 1 tenth to get 9 tenths altogether. She had 4 ones. The product is 4.972.

2. For example, 1.2 results in an estimate that is closer to the calculated product, and he can multiply 4 times 1.2 mentally.
3. **a)** For example, his estimate showed that the product was about 5 so the decimal point goes between the 4 and the 9 to result in the number closest to 5.
   b) The number of decimal places is the same.
   c) For example, the product of 1×4 is 4 and the product of 4×243 thousandths is close to 900 thousandths.
4. For example, they are the same because you are using the same digits. They are different because the place value of each digit is different.
5. **a)** For example, about 30; Round 5.695 to 6 and multiply by 5.
   b) For example, about 10.8; Round 1.185 to 1.2 and multiply by 9.
6. **a)** 26.241
   b) 47.448
7. **a)** For example, about 31; $7 \times 4 + 3 = 31$
   b) For example, about 21; $9 \times 2 + 3 = 21$
8. **a)** For example, about 14; 11.921
   b) For example, about 8; 7.664
   c) For example, about 18; 18.72
   d) For example, about 40; 41.48
9. Yes. For example, She is expressing 2.732 in expanded form and multiplying each part by 6.
10. 2.616 kg
11. 2.510 km^2
12. 8

2C Dividing Decimals by Whole Numbers (Direct Instruction)

Expectations

- [demonstrate an understanding of addition and subtraction of fractions and integers, and] apply a variety of computational strategies to solve problems involving whole numbers and decimal numbers
- solve problems involving the [multiplication and] division of decimal numbers to thousandths by one-digit whole numbers, using a variety of tools and strategies
- solve multi-step problems arising from real-life contexts and involving whole numbers and decimals, using a variety of tools and strategies
- use estimation when solving problems involving operations with whole numbers, decimals[, and percents,] to help judge the reasonableness of a solution
- evaluate expressions that involve whole numbers and decimals, including expressions that contain brackets, using order of operations

Answers

1. For example, Jody thought of 1.098 as about 1 or 10 tenths and estimated 10 divided by 9 as about 1.
2. **a)** For example, Jody estimated that the quotient was about 1 tenth of a centimetre so she placed the decimal point to the left of the 1 so that the quotient would be about 1 tenth.
   b) The number of decimal places is the same.
3. For example, it is the same because you are using the same digits. It is different because the place value of each digit is different.
4. $9 \times 0.122 = 1.098$. For example, multiplication and division are inverse operations.
5. **a)** For example, about 2.1; 2.132
   b) For example, about 0.3; 0.375
6. **a)** 4.198
   b) 4.146

7. **a)** For example, slightly greater than 1; Round down to 5 and divide by 5
   b) For example, about 2; Think of 19.584 as close to 18 and $18 \div 9 = 2$.
8. **a)** For example, about 2; 2.648
   b) For example, about 1.5; 1.783
9. **a)** 2.083; $2.083 \times 8 = 16.664$
   b) 4.713; $4.713 \times 6 = 28.278$
10. **a)** 10.811
    b) 1.053
11. 0.887 kg
12. 1.392 km^2
13. $1.329 + (1.329 \div 3) + (2 \times 1.329) = 4.430$; 4.430 kg

2D Solving Unit Rate Problems (Guided Activity)

Expectations

- demonstrate an understanding of proportional relationships, using [percent, ratio, and] rate
- use estimation when solving problems involving operations with whole numbers, decimals[, and percents,] to help judge the reasonableness of a solution
- demonstrate an understanding of rate as a comparison, or ratio, of two measurements with different units
- solve problems involving the calculation of unit rates

Answers

A. Divide the total cost by 4; $11.24
B. $12.36
C. $11.90
D. The Best Four
1. For example, it gives the price for each item. Then you can compare the prices per item.
2. **a)** The Best Four
   b) For example, Ryan might care what CDs are in the set, or may want to buy only two CDs.
3. For example, you divide the cost by the number of CDs in the set because you are calculating the rate for the cost of 1 CD or the cost per CD.
4. **a)** For example, about $0.50/kg; $0.55/kg
   b) For example, about 60 km/h; 64 km/h
5. 540 mL at $1.89 because $2.49 for 680 mL is about $0.00366/mL and 540 mL for $1.89 is $0.0035/mL.
6. **a)** For example, about 5 goals per game because $45 \div 9 = 5$
   b) For example, about $1/kg because $5 \div 5 = 1$
7. For example, Romona: about 10 km/h; Simon: about 10 km/h; Romona: 11.7 km/h, Simon:10.5 km/h; Romona bicycled faster.
8. $66.32 for 8 h
9. **a)** London 0.7°C, Thunder Bay 0.6°C
   b) London
10. **a)** $(3.25 \times 9) + (2.98 \times 6)$
    b) $47.13
11. $0.44
12. **a)** For example, John wants to buy motor oil for his boat. He wants to get the best price he can. Which should he buy? $9.68 for 8 L
    b) For example, John is selling motor oil for boats. He wants to make as much profit as he can. Which rate should he sell the oil at? $7.05 for 5 L

13. For example, she can divide $12.25 by 12 and $2.99 by 3 to calculate the rate per dozen.
14. $0.30

2E Dividing a Whole Number by a Decimal (Exploration)

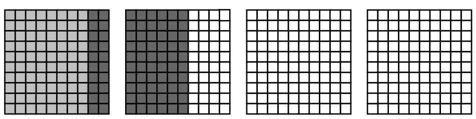
Expectations

- [demonstrate an understanding of addition and subtraction of fractions and integers, and] apply a variety of computational strategies to solve problems involving whole numbers and decimal numbers
- divide whole numbers by [simple fractions and] by decimal numbers to hundredths, using concrete materials
- solve multi-step problems arising from real-life contexts and involving whole numbers and decimals, using a variety of tools and strategies

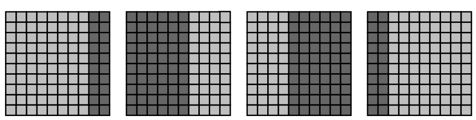
Answers

B. For example, about 5, Chang needs a little less than 1 m of framing for 1 picture so he can frame about 5 pictures with 4.0 m of framing.

C. For example, Chang coloured the remaining 2 columns of the first base ten block and 6 columns of the next base ten block to show 8 tenths or 0.8 m needed to frame the second picture.

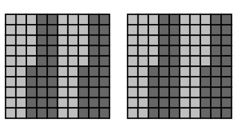


D. 5; For example, Chang used a different colour for each group of 8 columns since 8 columns represents 0.8 m, the amount needed for each picture. There are 5 groups of 8 columns or 5 groups of 0.8 so Chang can frame 5 pictures.

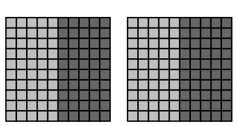


F. For example, about 8. It looks like Tynessa can frame 4 pictures with each metre so she can frame 8 with 2.00 m.

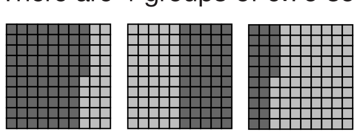
E., G. 8; For example, Tynessa used a different colour for each group of 25 hundredths or 2 columns for 2 tenths, or 20 hundredths, plus 5 squares in another column for 5 hundredths. This is the amount needed for each picture. There are 8 groups of 0.25, so Tynessa can frame 8 pictures.



1. For example, Chang's model shows 4 ones divided into sections so that each section represents 0.8. There are 5 sections or 5 groups of 0.8 to show that $4 \div 0.8 = 5$.
2. For example, Tynessa's model shows 2 ones divided into sections so that each section represents 0.25. There are 8 sections or 8 groups of 0.25 to show that $2 \div 0.25 = 8$.
3. **a)** 4; For example, the models show 2 wholes divided into groups with each group representing 0.5. There are 4 groups of 0.5 so the model shows $2 \div 0.5 = 4$. Miguel can frame 4 pictures.



b) 4; For example, the models show 3 wholes divided into groups with each group representing 0.75. There are 4 groups of 0.75 so the model shows $3 \div 0.75 = 4$. Kwami can frame 4 pictures.

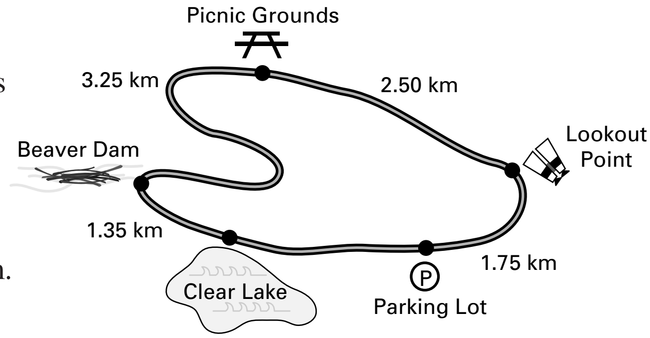


2A Adding and Subtracting Decimals Mentally

GOAL Solve problems by adding and subtracting decimals mentally.

Explore the Math

Fawn is hiking along a 9.50 km trail that starts and ends at a parking lot. She hiked 1.75 km to Lookout Point. Then she hiked 2.50 km farther and stopped at the picnic grounds. After hiking another 3.25 km, Fawn arrived at Beaver Dam where she read a sign saying the distance from Beaver Dam to Clear Lake is 1.35 km.



? How far is Clear Lake to the end of Fawn's hike?

A. Which two distances have a whole number sum? Add these distances mentally. Explain your thinking.

B. Using mental strategies, add the whole number parts of the other distances to the sum from step A.

C. Using mental strategies, add the decimal parts of these other distances to the sum from step B. How far will Fawn have hiked when she reaches Clear Lake?

D. Using mental strategies, tell how far Clear Lake is from the end of Fawn's hike. Explain your thinking.

E. Did you find the map helpful? Why or why not?

Reflecting

1. a) James calculated the distance from Clear Lake to the end of Fawn's hike by thinking 9.50 – 2.50, 1.75 + 3.25, 7 – 5, 2 – 1, 1 – 0.35. Describe his strategy. Explain how to finish his calculations mentally.

b) Make up your own strategy to calculate the distance from Clear Lake to the end of Fawn's hike mentally.

c) Do you prefer the strategy in steps A to D, James's strategy, or your own strategy? Explain your choice.

2. a) Simon hiked 4.7 km and 8.8 km. He calculated the total distance by thinking 4 + 8, 12 + 0.7, 12.7 + 0.3, 13 + 0.5. Describe his strategy.

b) Explain how to use Simon's strategy to calculate the total distance for 6.9 km and 2.4 km.

3. a) Jody hiked 5.8 km of a 9.3 km hike. She calculated the distance remaining by thinking 9.3 – 5, 4.3 – 0.3, 4 – 0.5. Describe her strategy.

b) Explain how to use Jody's strategy to calculate the distance remaining from 6.4 km after hiking 3.7 km.

4. a) Create an addition or subtraction problem with decimals that you can solve mentally.

b) Solve your problem mentally. Describe your strategy.

c) Trade problems with a classmate.

2B Multiplying Decimals by Whole Numbers (page 1)

GOAL Estimate and calculate products for multiplying decimal thousandths by a one-digit whole number.

You will need
- counters
- a decimal place value chart

Learn about the Math

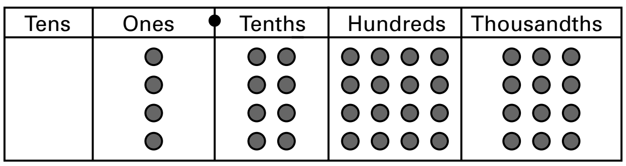
Rana is shipping a package of four identical soap stone carvings. The mass of each carving is 1.243 kg.

? What is the mass of all of Rana's carvings?

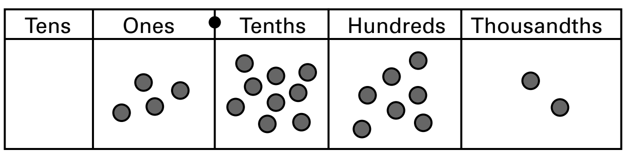
Example 1: Using counters to model decimal multiplication

Use counters on a place value mat to determine the product of 4×1.243 kg.

Rana's Solution



I modelled four groups of 1 one, 2 tenths, 4 hundredths, 3 thousandths with counters on a place value mat.



Then I regrouped the counters to represent the product.

The mass of the four carvings is 4.972 kg.

Example 2: Estimating and calculating a product

Estimate the product for 4×1.243 kg, and then calculate 4×1.243 kg.

Ryan's Solution

Estimate. 4×1.243 kg is about 4×1.2 kg is about 5 kg	I rounded 1.243 kg to 1.2 kg, and then multiplied. The product of 4×1.243 kg is about 5 kg.
Calculate.	I multiplied the whole numbers 4×1243.
$\begin{array}{r} 1.243 \\ \times \quad 4 \\ \hline 4.972 \end{array}$	My estimate shows the product is about 5, so I placed the decimal point between the 4 and 9. The mass of the four carvings is 4.972 kg.

Reflecting

1. Explain each step of combining and regrouping counters in Rana's solution.

2. Why do you think Ryan rounded 1.243 kg to 1.2 kg instead of 1 kg?

3. a) Explain how Ryan used his estimate to place the decimal point in the product.
   b) What do you notice about the number of decimal places in 1.243 kg and in 4.972 kg?
   c) Ryan says it makes sense that 4 times 1 one and 243 thousandths is 4 ones and 972 thousandths. Why does it make sense?

4. How is multiplying 4×1.243 the same as multiplying 4×1243? How is it different?

Multiplying Decimals by Whole Numbers (page 2)

A Checking

5. Estimate each product. Describe your strategies.

a) 5×5.695 **b)** 8×1.185

6. Choose a strategy and multiply.

a) 3×8.747 **b)** 6×7.908

B Practising

7. Estimate each product. Describe your strategies.

a) 7×4.528 **b)** 9×2.351

8. Estimate, and then multiply.

a) 7×1.703 **b)** 8×0.958 **c)** 3×6.24 **d)** 5×8.296

9. Kaitlyn says you can multiply 6×2.732 by calculating $6 \times 2 + 6 \times 0.7 + 6 \times 0.03 + 6 \times 0.002$. Do you agree? Explain.

10. A bag of marbles has a mass of 0.436 kg. What is the mass of six identical bags?

11. A park is 1.255 km by 2.000 km. What is the area?

C Extending

12. The thickness of a nickel is 0.176 cm. How many nickels are needed for a stack at least 1.4 cm high?

2C Dividing Decimals by Whole Numbers (page 1)

GOAL Estimate and calculate quotients for dividing decimal thousandths by a one-digit whole number.

Learn about the Math

A stack of nine identical dimes is 1.098 cm high.

? What is the thickness of each dime?

Example 1: Estimating and calculating a quotient

Estimate the quotient for 1.098 cm ÷ 9, and then calculate 1.098 cm ÷ 9.

Jody's Solution

Estimate. 1.098 ÷ 9 is about
10 tenths ÷ 9 is about 1 tenth

I rounded 1.098 cm to 1.0 cm or 10 tenths, and then divided.

The quotient for 1.098 cm ÷ 9 is about 1 tenth of a centimetre.

Each dime is about 1 tenth of a centimetre thick.

Calculate.

```
   0.122
9)1.098
   9
   --
   19
   18
   --
    18
    18
    --
     0
```

I divided the whole numbers 1098 ÷ 9.

My estimate shows the quotient is about 1 tenth so I placed the decimal point to the left of the 1.

I recorded 0 to the left of the decimal point.

The thickness of each dime is 0.122 cm.

I think my calculations are reasonable because a dime looks close to 0.122 cm.

Reflecting

1. Describe Jody's estimation strategy.

2. a) Explain how Jody used her estimate to place the decimal point in the quotient.

b) What do you notice about the number of decimal places in 1.098 cm and in 0.122 cm?

3. How is dividing 1098 ÷ 9 the same as dividing 1.098 ÷ 9? How is it different?

4. Show how to multiply to check Jody's division. How does your strategy show the relationship between multiplication and division?

Work with the Math

Example 2: Using the order of operations

Jason works at a summer camp that buys peanut butter in jars of 5.275 kg. To pack for camping trips, Jason used a scale that measures in thousandths of a kilogram. He put 0.956 kg in a red container, 1.238 kg in a blue container, and then put the remaining amount equally into three green containers.

Write an expression to determine the amount of peanut butter Jason put in each green container. Evaluate the expression.

Dividing Decimals by Whole Numbers (page 2)

Ravi's Solution

$(5.275 - 0.956 - 1.238) \div 3$	I know Jason started with 5.275 kg of peanut butter.
	He subtracted 0.956 kg for a red container, and then 1.238 kg for a blue container.
	He divided the amount left by 3.
$= 3.081 \div 3$ $= 1.027$	I did the calculations in the brackets first. Then I divided the result by 3.
	Jason put 1.027 kg of peanut butter in each green container.

A Checking

5. Estimate each quotient. Then divide.

a) $8.528 \div 4$ **b)** $2.625 \div 7$

6. Evaluate each expression.

a) $15.444 \div 6 + 0.232 \times 7$ **b)** $(1.709 + 0.364) \times 6 \div 3$

B Practising

7. Estimate each quotient. Explain your strategies.

a) $6.295 \div 5$ **b)** $19.584 \div 9$

8. Estimate, and then divide.

a) $5.296 \div 2$ **b)** $12.481 \div 7$

9. Divide. Then multiply to check.

a) $16.664 \div 8$ **b)** $28.278 \div 6$

10. Evaluate each expression.

a) $3.872 \times 3 - (4.025 \div 5)$ **b)** $0.412 \div 2 + 5.082 \div 6$

11. Jason divided 3.548 kg of peanut butter equally into four containers. How much is in each container?

12. A surveyor divided 8.352 km^2 of forest into six equal sections. What is the area of each section?

C Extending

13. Yuki and some friends collected rocks on a camping trip. Yuki has a rock with a mass of 1.329 kg. Paul has a rock with a mass that is a third of Yuki's and a rock that is twice as heavy as Yuki's. Write an expression for the total mass. Use the order of operations to calculate the mass.

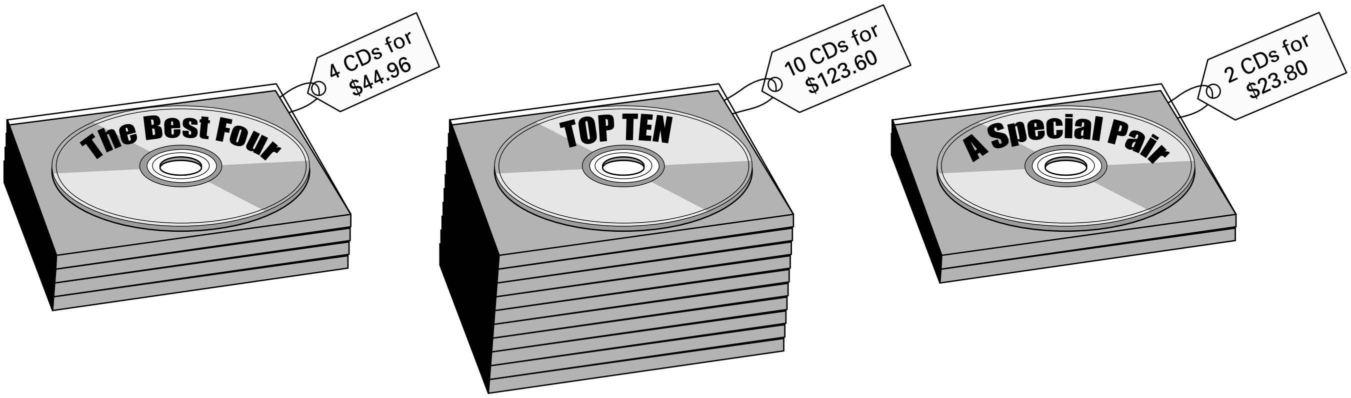
2D Solving Unit Rate Problems (page 1)

GOAL Determine and compare unit rates to solve problems.

You will need
- a calculator

Learn about the Math

Ryan is comparing prices of CDs. He wants to know the **unit rates** for different sets.



? In which set is the cost per CD the least?

unit rate
the rate for 1 unit, or the rate per unit

$\frac{\$5}{1 \text{ kg}}$ 65 km/h

Unit rates are written with the different units, with or without the 1.

A. How can you determine the cost per CD in The Best Four package? What is the cost per CD in the set for The Best Four?

B. What is the cost per CD in Top Ten?

C. What is the cost per CD in A Special Pair?

D. Which set has the least unit rate?

Reflecting

1. How is calculating the unit rate a strategy for comparing prices?

2. a) Which set of CDs is the best buy if Ryan wants to buy some CDs at as low a price as possible?

b) What might Ryan consider as well as the unit rate to decide which set to buy?

3. How do you know whether to divide the cost by the number of CDs in the set or the number of CDs in the set by cost?

Solving Unit Rate Problems (page 2)

Work with the Math

Example: Calculating the rate of speed

The 06:55 train from Toronto to Montreal travels 539 km in 5 h. The 17:55 train from Montreal to Quebec City travels 271 km in 3 h. The 17:30 train from Vancouver to Edmonton travels 1244 km in 23 h.

Which train travels fastest?

Bonnie's Solution

Estimate.

Toronto to Montreal: 539 km ÷ 5 h is about 550 km ÷ 5 h = 110 km/h

Montreal to Quebec City: 271 km ÷ 3 h is about 270 km ÷ 3 h = 90 km/h

Vancouver to Edmonton: 1244 km ÷ 23 h is about 1200 km ÷ 20 h = 60 km/h

I rounded, and then divided.

The speed for the train from Toronto to Montreal is about 110 km/h.

The speed for the train from Montreal to Quebec City is about 90 km/h.

The speed for the train from Vancouver to Edmonton is about 60 km/h.

Calculate.

I calculated each unit rate by dividing the distance by the number of hours.

Toronto to Montreal:
539 km ÷ 5 h = 107.8 km/h

The speed from Toronto to Montreal is 107.8 km/h. This is close to my estimate.

Montreal to Quebec City:
271 km ÷ 3 h = 90.333 … km/h

The speed from Montreal to Quebec City is 90.3 km/h, to the nearest tenth. This is close to my estimate.

Vancouver to Edmonton:
1244 ÷ 23 h = 54.086 … km/h

The speed from Vancouver to Edmonton is 54.1 km/h, to the nearest tenth. This is close to my estimate.

The train from Toronto to Montreal is fastest.

A Checking

4. Estimate, and then calculate each unit rate. Use your estimate to check.

a) 6 kg of bananas for $3.30 **b)** driving 448 km in 7 h

5. Which is a better buy for the same spaghetti sauce: 680 mL for $2.49 or 540 mL for $1.89? Use unit rates to explain your answer.

B Practising

6. Estimate each unit rate. Explain your estimation strategy.

a) 46 goals in 9 games **b)** $4.98 for 5 kg

7. Romona bicycled 35 km in 3 h. Simon bicycled 21 km in 2 h. Estimate each unit rate. Then calculate each unit rate to the nearest tenth. Who bicycled faster?

Solving Unit Rate Problems (page 3)

8. Kaitlyn's sister is researching rates of pay for summer jobs at stores. Which is the best rate of pay: \$47.90 for 6 h, \$66.32 for 8 h, or \$56.63 for 7 h?

9. In London, the temperature increased by 5.6°C in 8 days. In Thunder Bay, it increased from 12.4°C to 16.6°C in 7 days.

a) Calculate the unit rates.

b) Which city had a greater increase in temperature?

10. Sandra bought 9 m of satin ribbon that costs \$3.25/m and 6 m of velvet ribbon that costs \$2.98/m.

a) Write an expression to use the order of operations to calculate the total cost of the ribbon.

b) Evaluate the expression to the nearest cent.

11. Tickets for a concert cost \$9.00 each or a book of five tickets for \$42.80. How much less is the price per ticket in the book?

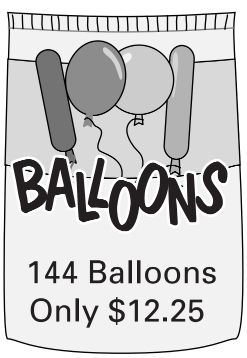
12. Mei is comparing these rates: \$3.87 for 3 L, \$7.05 for 5 L, and \$9.68 for 8 L.

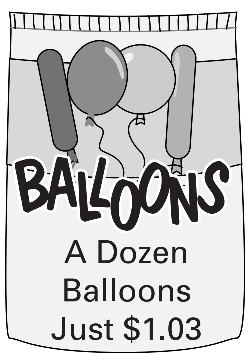
a) Create and solve a problem where you need the least unit rate for the solution.

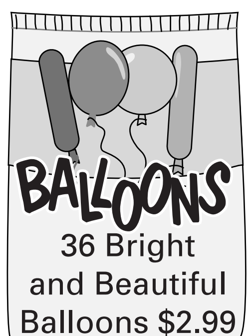
b) Create and solve a problem where you need the greatest unit rate for the solution.

C Extending

13. Indira is comparing prices of the same balloons sold in different packages. She says she can compare prices by dividing only two of the costs. Explain how.







14. A package of 16 prizes costs \$23.98, but is on sale for 20% off. How much would Kwami save per prize with the sale? Give your answer to the nearest cent.

2E Dividing a Whole Number by a Decimal

GOAL Use estimation and models to divide whole numbers by decimal tenths and by decimal hundredths.

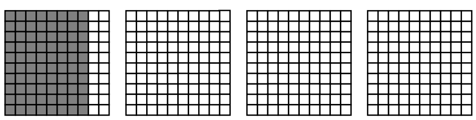
You will need
- paper base ten blocks
- scissors
- coloured pencils

Explore the Math

Chang has 4.0 m of framing. He needs 0.8 m to frame each picture. Tynessa has 2.00 m of framing. She needs 0.25 m to frame each picture.

? How many pictures can Chang frame and how many can Tynessa frame?

A. Chang cut out four base ten blocks to model 4.0 m of framing. He coloured 8 columns of squares to model the amount of framing for the first picture. Cut out and colour base ten blocks to match Chang's model.



B. Estimate how many frames Chang can frame. Explain your estimation strategy.

C. Chang used a different colour for the two remaining columns of the same base ten block and six columns of the next base ten block to model the framing for a second picture. Show this step with your base ten blocks. How does this model the framing for a second picture?

D. Complete Chang's model. How many pictures can Chang frame? How does your model show this?

E. Tynessa cut out two base ten blocks to model her 2.0 m of framing. She coloured two columns and half of another column of squares to model the amount of framing for the first picture. Colour base ten blocks to show this.

F. Estimate how many frames Tynessa can frame. Explain your estimation strategy.

G. Use different colours to complete Tynessa's model. How many pictures can Tynessa frame? How does your model show this?

Reflecting

1. Explain how Chang's model shows $4 \div 0.8 = 5$.

2. Explain how Tynessa's model shows $2 \div 0.25 = 8$.

3. Use a model to determine how many pictures each can frame.
Explain your thinking.

a) Miguel has exactly 2 m of framing. She needs 0.5 m to frame each picture.

b) Kwami has exactly 3 m of framing. He needs 0.75 m to frame each picture.

Chapter 3 Expectations and Answers

3A Circle Graphs (Direct Instruction)

Expectations

- select and justify the most appropriate representation of a quantity (i.e., fraction, decimal, percent) for a given context
- collect and organize categorical, discrete [or continuous] primary data and secondary data and display using charts and graphs, including relative frequency tables and circle graphs
- make and evaluate convincing arguments, based on the analysis of data
- collect data by conducting a survey [or an experiment] to do with themselves, their environment, issues in their school or community, or content from another subject and record observations [or measurements]
- select an appropriate type of graph to represent a set of data, graph the data using technology, and justify the choice of graph (i.e., from types of graphs already studied)
- read, interpret, and draw conclusions from primary data and from secondary data presented in charts, tables, and graphs (including relative frequency tables and circle graphs)
- make inferences and convincing arguments that are based on the analysis of charts, tables, and graphs
- research and report real-world applications of probabilities expressed in fraction, decimal, and percent form

Answers

1. **a)** For example, I can use a protractor to measure 180° in half the circle and another 180° in the other half of the circle.

b) For example, the sections of the graph go completely around the circle.

c) For example, 16 : 60 = 96 : 360 so the number of degrees out of 360° at the centre of the circle for the section representing the choice *fraction* is 96°.

d) For example, I can add number of degrees for all the other choices and subtract the sum from 360°.

2. For example, yes. The numbers show how many people made each choice and you can add the numbers to find out how many people were asked.
For example, no. The sizes of the sections of the circle graph show the comparison for the choices.

3. For example, the whole is the number of people in the survey, and the part is the number of people who made each choice.

4. **a)** No. For example, the times are not part of a whole. A bar graph can be used.

b) Yes. For example, provinces and territories are part of the whole of Canada.

c) No. For example, temperature is not part of a whole. A broken-line graph or a bar graph can be used.

5. For 45, 135°; for 8, 24°; for 50, 150°; for 17, 51°

6. **b)** The circle graph will represent the data in the survey.

c) For example, according to my survey, more people chose *percent*.

7. **a)** For example, Should students be required to wear uniforms?

b) The circle graph will represent the data in the survey.

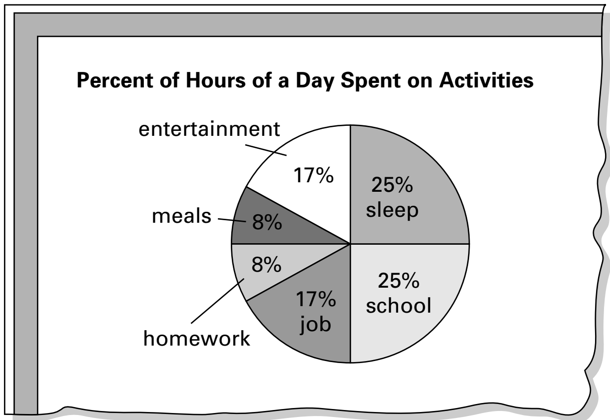
c) For example, How does the data in the graph compare with your opinion?

8. **b)** For example, percent

c) For example, yes, because percents emphasize that it is a number out of 100.
For example, no, because fractions show it is part of a whole.

d) Yes, because you are displaying data that is now part of a whole.

9. For example,



b) For example, most students spent half of their day either sleeping or going to school. The graph shows this because 25% plus 25% equals 50% which is equivalent to half.

3B Central Tendency (Exploration)

Expectations

- make and evaluate convincing arguments, based on the analysis of data
- determine, through investigation, the effect on a measure of central tendency (i.e., mean, median, and mode) of adding or removing a value or values
- research and report real-world applications of probabilities expressed in fraction, decimal, and percent form

Answers

A. For example, I agree. The median would not be changed if you removed the greatest and least values. The mode would be changed if the values removed are the mode. The mean would probably be changed if you remove either the greatest or the least because it is calculated by adding all the values and dividing by the number of values.

B. Mean about 51%, median 50%, mode 30% and 50%

C. Mean about 49%, median 50%, mode 50%

D. Mean about 49%, median 50%, mode 50%

E. Mean about 38%, median 40%, mode 50%

F. Mean, median, and mode will be depend on students' researched data.

1. a) For example, it changed the mean because the numbers added to determine the total and the number of percents are different. It did not change the median because the values removed did not affect the middle value. It changed the mode because the percents for the last 2 days are 50% and 20%. With the additional day of 50% included, 50% becomes the only mode.

b) For example, it did not change the mean, median, or mode because, in this case, the effect of removing the least value was balanced by removing the greatest value.

c) For example, it changed the mean because the numbers added to determine the total and the number of percents are different. It changed the median because the values removed affected which is the middle value. It did not change the mode because the mode was not one of the values removed.

2. For example, I think it would almost always change the mean because each value is added when calculating the mean. It would often change the median because which value is the middle is easily changed by removing greatest or least values. It would sometimes change the mode, but frequently the mode is not one of the values added or removed so it would stay the same. The effect depends on whether the values removed are the least and greatest values, the greatest values only, or the values determined by the situation.

3. Percent. For example, yes, because percents make it easy to compare predictions from one day to the next.

3C Analyzing Misleading Graphs (Exploration)

Expectations

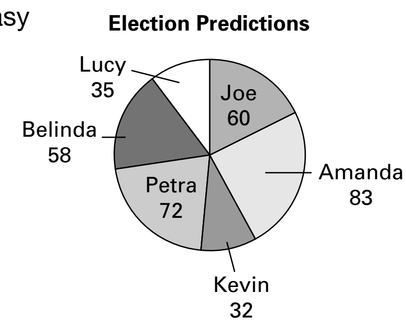
- [collect and] organize categorical, discrete[, or continuous primary data and] secondary data and display using [charts and] graphs, including [relative frequency tables and] circle graphs
- make and evaluate convincing arguments, based on the analysis of data
- [collect and] organize categorical, discrete[, or continuous primary data and] secondary data and display the data in [charts, tables, and] graphs (including [relative frequency tables and] circle graphs) that have appropriate titles, labels and scales that suit the range and distribution of the data, using a variety of tools
- select an appropriate type of graph to represent a set of data, graph the data using technology, and justify the choice of graph (i.e., from types of graphs already studied)
- read, interpret, and draw conclusions [from primary data and] from secondary data presented in charts, tables, and graphs (including relative frequency tables and circle graphs)
- identify, through investigation, graphs that present data in misleading ways
- make inferences and convincing arguments that are based on the analysis of charts, tables, and graphs

Answers

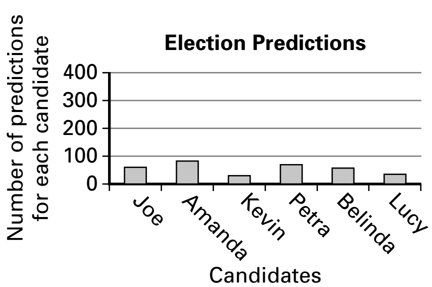
A. For example, they display the same data. The vertical scales start at different values.

B. No. For example, since the vertical scale of the second graph starts at 30 instead of 0, it gives the illusion that the election race is not close.

C. For example, this circle graph makes it easy to compare the number of predictions for each candidate.



D. For example, the vertical scale goes to a value much greater than the greatest data value so it looks as though the predictions are much closer than they are.



1. a) For example, if the least value on a scale is too close to the least data value, the graph may be misleading because it can look as though the data are much farther apart than they actually are.

b) For example, if the greatest value on a scale is much greater than the greatest data value, the graph may be misleading because it can look as though the data are much closer than they actually are.

2. For example, the graph shown to represent the government party's popular support. Depending on what newspaper it was, it showed a different party in the lead. The actual number of people interviewed was very small.

3. For example, someone might want to use the data for their own personal gain. They could want to make it look as though a candidate was in the lead to affect the number of people who would vote.

4. You can look at the data being presented, the type of graph being used, and decide whether the type of graph and the scale are appropriate to display the data realistically.

3A Circle Graphs (page 1)

GOAL Conduct surveys, organize the data in a circle graph, and draw conclusions about the data.

You will need
- a compass
- a protractor
- a calculator
- data sources including newspapers, books, and the Internet

Learn about the Math

Romona read an advertisement for a lottery saying that the probability of winning a prize is 20%. She conducted a survey of 60 people asking, "Which do you think is the best way to express the probability of winning the lottery: as the percent 20%, as the decimal 0.20, as the fraction $\frac{20}{100}$, or it doesn't matter?"

Way of expressing probability	Tally for Preferences	Frequency
As a percent	𝍸 𝍸 𝍸 𝍸	20
As a decimal	𝍸 𝍸 𝍸	15
As a fraction	𝍸 𝍸 𝍸 |	16
It doesn't matter	𝍸 ||||	9

? How can Romona display the data as a circle graph?

Example 1: Using ratios to construct a circle graph

Construct a circle graph to display Romona's data.

Romona's Solution

$20 : 60 = \blacksquare : 360$

I wrote a ratio for the number of people out of 60 who chose *percent*. I know the number of degrees at the centre of a circle is 360°, so I want to calculate the missing term in $\blacksquare : 360$ for a ratio equivalent to $20 : 60$.

$\times 6$

$20 : 60 = \blacksquare : 360$

$\times 6$

Since $60 \times 6 = 360$, I multiplied 20 by 6 to get the missing term in the second ratio.

$20 : 60 = 120 : 360$

$15 : 60 = \blacksquare : 360$

$15 : 60 = 90 : 360$

I calculated an equivalent ratio for the choice *decimal* by multiplying 15 by 6 to get the missing term in the second ratio.

$16 : 60 = \blacksquare : 360$

$16 : 60 = 96 : 360$

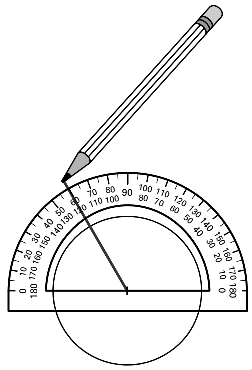
I calculated an equivalent ratio for the choice *fraction* by multiplying 16 by 6 to get the missing term in the second ratio.

$9 : 60 = \blacksquare : 360$

$9 : 60 = 54 : 360$

I calculated an equivalent ratio for the choice *doesn't matter* by multiplying 9 by 6 to get the missing term in the second ratio.

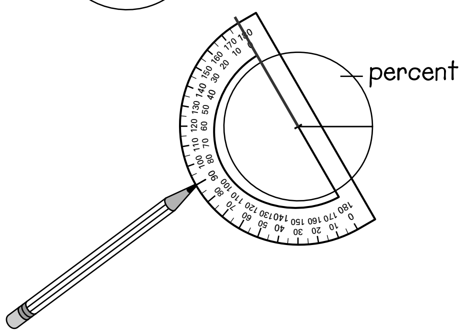
Circle Graphs (page 2)



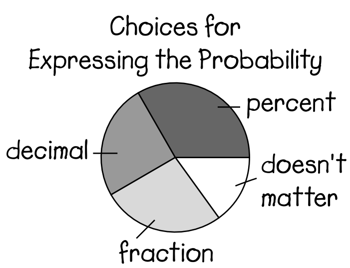
I drew a circle and marked the centre.

Then I drew a segment from the circle to the centre.

Since the ratio 120 : 360 is equivalent to the ratio 20 : 60 for the number of people who chose *percent*, I drew an angle of 120° at the centre. Next I'll label this part of the circle *percent*.



Since the ratio 90 : 360 is equivalent to the ratio 15 : 60 for the number of people who chose *decimal*, I drew an angle of 90° next to the angle of 120° for percent. Next I'll label this part of the circle *decimal*.



I drew an angle of 96° for the number of people who chose *fraction*, and then I recorded the title *fraction*.

I recorded *doesn't matter* in the remaining part of the circle.

Then I recorded the title for the graph.

Reflecting

1. a) How can you check by measuring that the number of degrees at the centre of a circle is 360º?

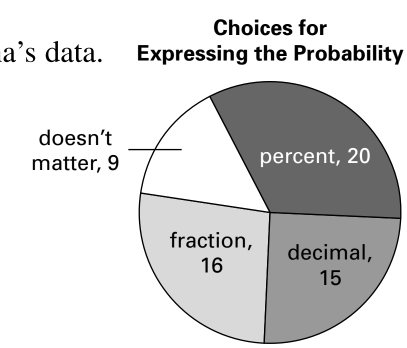
b) How can you use Romona's method to explain how you knew that 360º is the sum of 120º, 90º, 96º, and 54º without adding?

c) Why did Romona draw an angle of 96º for the section of the circle for *fraction*?

d) How do you know, without measuring, the angle measure of the section of the graph for *doesn't matter*?

2. James used a software program to construct a circle graph for Romona's data. Do you think the numbers are useful on the graph? Why or why not?

3. A circle graph is appropriate for displaying data that are part of a whole. Look at the circle graph on the right. Explain how Romona's data are part of a whole.



Circle Graphs (page 3)

Work with the Math

Example 2: Making survey and graphing decisions

Colin's class is choosing topics to conduct a survey in their school and display the data in a circle graph. What topics might they choose?

Students' Solutions

Topic Ideas	Reasons for or against using topics
Miguel: "We can conduct a survey asking students to choose from a list of suggestions for a school project on the environment."	The data can be displayed in a circle graph because the number of students who make each choice can be expressed as a ratio of the number surveyed.
Kwami: "We can conduct a survey where students choose noon-hour activities to be offered in our cafeteria."	A circle graph can display the ratio for the number of students who make each choice compared with the number of students asked.

A Checking

4. Tell whether each topic is appropriate for representing in a circle graph. Justify your answers. If a circle graph is not appropriate, name a type of graph that can be used.

a) the time for each runner in a track event

b) the population in each province and territory

c) the temperature each day of the week

5. Ravi conducted a survey of 120 students. The number of students who made each choice is 45, 8, 50, and 17. What measure does he need for each angle at the centre of a circle graph?

B Practising

6. a) Use Romona's question to conduct a survey of 30 students.

b) Construct a circle graph for the results of your survey.

c) State a conclusion based on your graph. Explain how your graph supports your conclusion.

7. a) Make up your own survey question about an issue in your school or community.

b) Conduct the survey. Represent the results in a circle graph.

c) Create a question that can be answered from your graph. Trade questions.

8. a) Research the probability of winning a lottery.

b) Is the probability expressed as a fraction, a decimal, or a percent?

c) Do you think this is the best way to express the probability? Why or why not?

d) Suppose you researched the probability of winning five different lotteries. Would the data be appropriate for displaying in a circle graph? Why or why not?

C Extending

9. a) Find a circle graph in a newspaper or reference book or on the Internet.

b) Write a conclusion based on the graph. Explain how the graph supports your conclusion.

3B Central Tendency

GOAL Compare effects on measures of central tendency of adding or removing values.

You will need
- a calculator
- a data sources for weather predictions

Explore the Math

Each day for two rainy weeks in April, Simon researched the prediction about the probability of precipitation from a Web site.

Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
75%	40%	90%	50%	100%	20%	70%
0%	60%	30%	30%	50%	50%	20%

Simon thinks that the effect of removing the greatest and least values will not have the same affect on the mean, the median, and the mode.

? How can you compare the effect of removing values?

A. Predict. Do you agree with Simon? Justify your prediction.

B. Determine the mean, median, and mode for the first 12 days of Simon's data. Round the mean to the nearest percent.

C. Repeat step B for all Simon's data.

D. Remove the least and greatest values from all of Simon's data, and repeat step B.

E. Remove the three greatest values from all of Simon's data and repeat step B.

F. Research predictions for the probability of precipitation for as many days as possible. Repeat steps B to E for your data.

Reflecting

1. a) Did including the last two days of Simon's data affect the measures of central tendency? Why or why not?

b) Did removing the least and greatest values from all of Simon's data affect the measures of central tendency? Why or why not?

c) Did removing the three greatest values from all of Simon's data affect the measures of central tendency? Why or why not?

2. Do you think adding or removing values from data would usually affect the measures of central tendency? Justify your answer.

3. Is the data you researched expressed as a fraction, a decimal, or a percent? Do you think this is the best way to express the data? Why or why not?

3C Analyzing Misleading Graphs

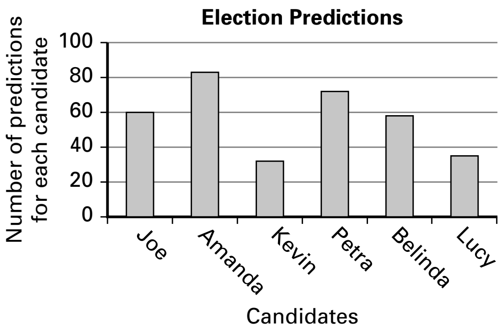
GOAL Identify graphs that are misleading, and explain how they are misleading.

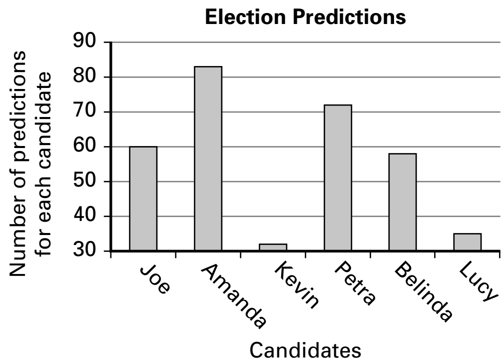
You will need
- a graphing program or a ruler, centimetre grid paper, a compass, a protractor
- data sources including newspapers, books, and the Internet

Explore the Math

Indira conducted a survey asking students at her school who they thought would win an election.

Candidate	Number of people predicting the candidate will win
Joe	60
Amanda	83
Kevin	32
Petra	72
Belinda	58
Lucy	35





? How can graphs be misleading?

A. How are Indira's graphs the same? How are they different?

B. Do you think each graph displays the data accurately? Explain.

C. Construct your own graph to display Indira's data appropriately. Justify your choice.

D. Construct your own graph to display Indira's data in a way that is misleading. Experiment with the effect of increasing and decreasing the least and greatest values on the scale. Choose a misleading graph. How is it misleading?

Reflecting

1. a) How can a graph make it appear that the data vary more than they actually do?

b) How can a graph make it appear that the data are closer than they actually are?

2. Research graphs in the media or in a data source. Are any misleading? If so, explain how one graph is misleading. If not, choose a graph, and explain how it could have been constructed differently to make it misleading.

3. Why do you think someone might want to create a misleading graph?

4. How can you analyze a graph to decide whether it is misleading?

Chapter 5 Expectations and Answers

5A Solving Area Problems (Direct Instruction)

Expectations

- report on research into real-life applications of area measurements
- research and report on real-life applications of area measurements
- solve problems that require conversion between metric units of measure
- solve problems that require conversion between metric units of area (i.e., square centimetres, square metres)
- solve problems involving the estimation and calculation of the area of a trapezoid
- estimate and calculate the area of composite two-dimensional shapes by decomposing into shapes with known area relationships

Answers

1. For example, Kaitlyn realized that by using the formula for the area of a trapezoid she could multiply the sum of the lengths of the parallel sides by the height and divide by 2. She changed the order of the steps to think of dividing the sum of the lengths of the parallel sides by 2, and then multiplying by the height. Since 7 m is about halfway between 6.50 m and 7.60 m, she replaced the step of adding the lengths of the parallel sides and dividing by 2 with 7 m.

2. 32.89 m^2; My answer is reasonable because it is close to Kaitlyn's estimate which is 32 m^2.

3. For example, when the area is in square metres, the numbers are less than when the area is in square centimetres. It is easier to work with the area in square metres.

4. For example, if I am going to build something such as a club house or paint my room, it is important that I research to make sure that I have everything I need and that I have the correct amount of each kind of material. I could use the Internet, visit a carpenter, or use books from the library. It is important to do this so that it is built properly and safely.

5. **a)** 60 000 cm^2
b) 47 500 cm^2

6. **a)** For example, for the rectangle, convert 2.4 cm to 24 mm. Round 24 mm to 20 mm and 75 mm to 80 mm. Multiply 20 mm by 80 mm to get 1600 mm^2. For the trapezoid, convert 1.9 cm to 19 mm. Round 19 mm to 20 mm, 75 mm to 80 mm, and 86 mm to 90 mm. Add 20 mm + 80 mm for 100 mm. Multiply 100 mm by 90 mm and divide by 2 to get 4500 mm^2. Add the areas 1600 mm^2 + 4500 mm^2 to get the estimate about 6100 mm^2.
b) 5842 mm^2

7. **a)** 55.2 m^2
b) 0.9307 m^2

8. **a)** 11 371 cm^2
b) 1.14 m^2

9. **a)** For example, about 27 m^2
b) 256 578 cm^2; 256 578 cm^2 = 25.66 m^2 to the nearest hundredth. This is close to my estimate 27 m^2.

10. For example, using the Internet, I researched how to build a skateboard. I must design the skateboard first so I can correctly determine how much wood I will need as well as the right amount of grip tape needed for my board. To do this I will calculate the area of the deck part of the skateboard. I will also need to determine what kind of hardware I will use. I will have to figure out what wheels I will use.

11. **a)** For example, about 1700 mm^2
b) 1958 mm^2

12. 30 000 : 7708

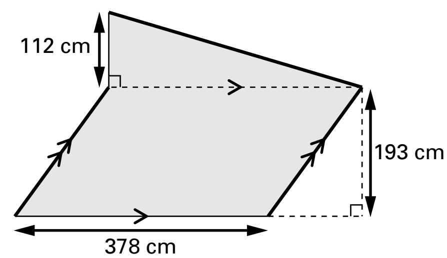
5A Solving Area Problems (page 1)

GOAL Estimate and calculate area, and convert units to solve problems.

You will need
- a calculator

Learn about the Math

Kaitlyn is painting this design on a stage for a drama festival. The stage is a trapezoid with parallel sides with lengths 6.50 m and 7.60 m. The parallel sides are 6.00 m apart.



? About what area of the stage is not covered by Kaitlyn's design?

Example: Estimating areas and the difference between the areas

Estimate the area of the stage and the area of Kaitlyn's design.
Express both in the same unit, and estimate the difference.

Kaitlyn's Solution

Area of a parallelogram $= b \times h$ $= 378$ cm $\times$ 193 cm is about 400 cm $\times$ 200 cm	First I estimated the area of Kaitlyn's design. I rounded measurements of the parallelogram part of the design and estimated the area.
$= 80\ 000$ cm^2	The area of the parallelogram is about 80 000 cm^2.
Area of a triangle $= b \times h \div 2$	I rounded measurements of the triangle part of the design and estimated the area.
$= 378$ cm $\times$ 112 cm $\div$ 2 is about 400 cm $\times$ 100 cm $\div$ 2	Since the opposite sides of a parallelogram are equal, the base of the triangle is 378 cm.
$= 20\ 000$ cm^2	The area of the triangle is about 20 000 cm^2.
20 000 cm^2 + 80 000 cm^2 $= 100\ 000$ cm^2	The total area of the design is about 100 000 cm^2.
Area of a trapezoid $= (a + b) \times h \div 2$	Then I estimated the area of the stage. I figured out that 7 m is about halfway between 6.50 m and 7.60 m.
$= (6.50$ m $+ 7.60$ m$) \times 6.00$ m $\div 2$ is about 7 m $\times$ 6 m $= 42$ m^2	The area of the stage is about 42 m^2.
1 m^2 = 100 cm $\times$ 100 cm = 10 000 cm^2	1 m = 100 cm and 1 m^2 = 1 m $\times$ 1 m.
100 000 cm^2 = (100 000 $\div$ 10 000) m^2 = 10 m^2	To express the estimated area of Kaitlyn's design in square metres, I divided its estimated area by 10 000.
42 m^2 – 10 m^2 = 32 m^2	I subtracted my estimate for the area of Kaitlyn's design from my estimate for the area of the stage. The area of the stage that is not covered by Kaitlyn's design is about 32 m^2.

Solving Area Problems (page 2)

Reflecting

1. Explain Kaitlyn's strategy for estimating the area of the stage.

2. Calculate the area of the stage that is not covered by Kaitlyn's design to the nearest hundredth of a square metre. Explain how to use Kaitlyn's estimate to determine whether your calculations are reasonable.

3. Why do you think Kaitlyn converted the area of the design to square metres instead of converting the area of the stage to square centimetres?

4. Why might you want to calculate or estimate area in real life? How could you research the information you would need?

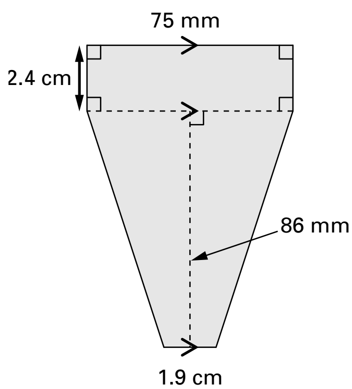
A Checking

5. Express each area in square centimetres.

a) $6\ m^2$ **b)** $4.75\ m^2$

6. a) Estimate the area in square millimetres. Explain your strategy.

b) Calculate the area in square millimetres. Use your estimate to check.



B Practising

7. Express each area in square metres.

a) $552\ 000\ cm^2$ **b)** $9307\ cm^2$

8. Ryan has $2.00\ m^2$ of material. He wants a rectangular piece 88 cm by 73 cm and a triangular piece with a base of 42 cm and a height of 1.05 m to make a costume.

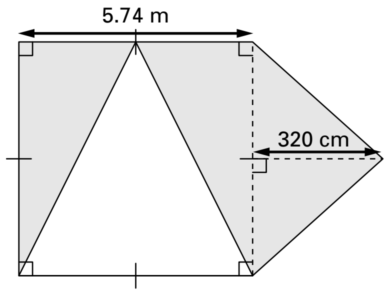
a) How many square centimetres will be left?

b) Express the number of square centimetres left as square metres to the nearest hundredth.

Solving Area Problems (page 3)

9. a) Estimate the area of the shaded part in square metres.

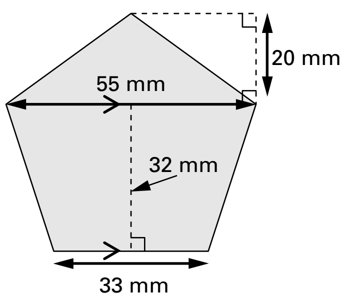
b) Calculate the area of the shaded part in square centimetres.
Explain how your estimate shows whether your calculation is reasonable.



10. Choose a situation such as building a skateboard or painting a porch. Research information you would need. Write a few sentences to report on what you researched and why you would need the information. Use math language.

C Extending

11. Estimate, and then calculate the area of this regular pentagon.



12. A stage prop consists of a trapezoid and a triangle. The trapezoid has parallel sides 2.50 m and 3.50 m long that are 1.00 m apart. The base of the triangle is 164 cm and its height is 94 cm. What is the ratio of the area of the trapezoid to the area of the triangle?

Chapter 7 Expectations and Answers

7A Sorting Triangles and Quadrilaterals (Exploration)

Expectations

- [construct related lines, and] classify triangles, quadrilaterals[, and prisms]
- sort and classify triangles and quadrilaterals by geometric properties related to symmetry, angles, and sides, through investigation using a variety of tools and strategies

Answers

B. The triangles in the overlap have exactly two equal angles and they have line symmetry. They are isosceles triangles. The triangles that are not in either circle do not have either exactly two equal angles or line symmetry.

C. For example, one circle is *At least two equal sides* and the other circle is *Right angles*. The triangles in the overlap are right isosceles triangles. The triangles that are not in either circle are obtuse scalene triangles or acute scalene triangles.

E. The quadrilaterals in the overlap have opposite sides that are equal and parallel and all right angles. They are rectangles and squares. The quadrilaterals that are not in either circle do not have opposite sides that are equal and parallel and they do not have right angles. They can be trapezoids or irregular quadrilaterals.

F. For example, one circle is *Line symmetry* and the other circle is *At least two equal sides*. The quadrilaterals in the overlap have line symmetry and at least two equal sides. Squares, rectangles, and rhombi are in the overlap. The quadrilaterals that are not in either circle do not have line symmetry or any equal sides. They are irregular quadrilaterals.

1. Venn diagrams helped me organize triangles and quadrilaterals with common properties of line symmetry, equal angles, right angles, acute angles, obtuse angles, equals sides, and parallel sides. I could classify the shapes according to the properties, for example, right triangles, rectangles, or parallelograms.

7B Relationships for Congruent Shapes (Exploration)

Expectations

- determine, through investigation using a variety of tools, relationships among area, perimeter, corresponding side lengths, and corresponding angles of congruent shapes

Answers

B. The corresponding angles are equal.

C. The corresponding sides are equal.

D. The perimeters are equal.

E. The areas are equal.

F. In congruent quadrilaterals, the corresponding angles are equal, the corresponding sides are equal, the perimeters are equal, and the areas are equal.

1. No. For example, the orientation of the shapes does not affect the congruency of the shapes.

2. **a)** Corresponding sides would be equal, and corresponding angles would be equal. For example, a reflection image is congruent to the pre-image. A rotation image is congruent to the pre-image. You are only changing the position of the figure, not the shape or size .

b) The perimeters would be equal. The areas would be equal.

3. **a)** False. For example, a triangle with a base of 6 cm and height of 2 cm has the same area as a triangle with a base of 4 cm and a height of 3 cm, but they are not congruent.

b) True. For example, corresponding sides of congruent figures have equal lengths so the perimeters are equal.

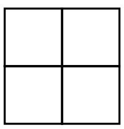
c) False. For example, a triangle with sides 2 cm, 3 cm, and 4 cm has the same perimeter as a triangle with three sides each 3 cm but the corresponding sides are not of equal length.

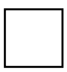
7C Dilatations with Pattern Blocks (Exploration)

Expectations

- develop an understanding of similarity, and distinguish similarity and congruence
- [describe location in the four quadrants of a coordinate system,] dilatate two dimensional shapes, and apply transformations to create and analyse designs
- demonstrate an understanding that enlarging or reducing two-dimensional shapes creates similar shapes
- distinguish between and compare similar shapes and congruent shapes, using a variety of tools and strategies
- identify, perform, and describe dilatations (i.e., enlargements and reductions), through investigation, using a variety of tools
- create and analyse designs involving [translations, reflections,] dilatations[, and/or simple rotations] of two-dimensional shapes, using a variety of tools and strategies

Answers

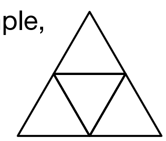
A. 

B. 

C. They are the same shape.

D. It is similar because it has the same shape as a trapezoid pattern block. It is an enlargement. It is made of four trapezoid pattern blocks.

E. No. They are not the same size. For example,



1. **a)** Yes. They would be the same shape.
   b) Reduction. For example, it is smaller than the pre-image which is smaller than Jody's design.
2. Trapezoids 1, 3, and 5 are not the same shape as a trapezoid pattern block so they cannot be dilatations of a trapezoid pattern block.

7D Dilatations (Direct Instruction)

Expectations

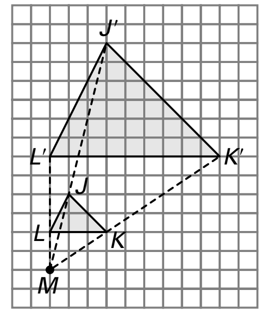
- develop an understanding of similarity, and distinguish similarity and congruence
- [describe location in the four quadrants of a coordinate system,] dilatate two dimensional shapes, and apply transformations to create and analyse designs
- demonstrate an understanding that enlarging or reducing two-dimensional shapes creates similar shapes
- distinguish between and compare similar shapes and congruent shapes, using a variety of tools and strategies
- identify, perform, and describe dilatations (i.e., enlargements and reductions), through investigation, using a variety of tools
- create and analyse designs involving [translations, reflections,] dilatations[, and/or simple rotations] of two-dimensional shapes, using a variety of tools and strategies

Answers

1. **a)** Yes.
   b) enlargement
   c) farther
   d) Yes. For example, when the scale factor is greater than 1, the pre-image is magnified to create the image. The size is increased and the image is moved farther from the dilatation centre. The shape remains the same so the image is similar to the pre-image.

2. The properties that are the same are shape and orientation. A property that is different is size.

3. For example, $\Delta J'K'L'$ is a dilatation of ΔJKL. The dilatation centre is point M. Point J′ is 3 times the distance from dilatation centre M as point J. Each of the points K′ and L′ is 3 times the distance from dilatation centre M as points K and L. So the scale factor is 3. $\Delta J'K'L'$ is similar to ΔJKL and it is an enlargement of ΔJKL.

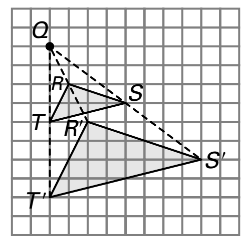


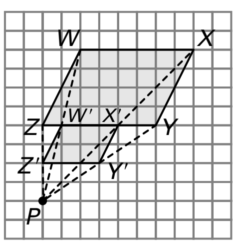
4. **B.** $\frac{1}{4}$

   C. $\frac{1}{2}$

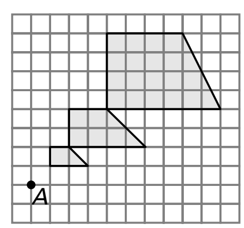
   E. $\frac{1}{5}$ When the scale factor is less than 1, the image is a reduction.

5. $\Delta R'S'T'$ is a dilatation of ΔRST. The dilatation centre is point Q. Point R′ is 2 times the distance from dilatation centre Q as point R. Each of the points S′ and T′ is 2 times the distance from dilatation centre Q as points S and T. So the scale factor is 2. $\Delta R'S'T'$ is similar to ΔRST and it is an enlargement of ΔRST.

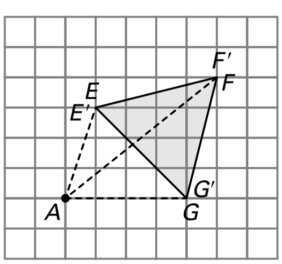


6. 

7. For example, the trapezoids are similar. The smallest trapezoid is enlarged as a result of a dilatation with the dilatation centre A and a scale factor of 2 to create the middle trapezoid. The largest trapezoid is a result of a dilatation of the smallest trapezoid with centre of dilatation A and a scale factor of 4. If lines were drawn joining vertices of the largest trapezoid to the centre of dilatation, the lines would pass through the corresponding vertices of the similar trapezoids.



8. Yes; yes. Point E′ is the image of point E. Point F′ is the image of point F. Point G′ is the image of point G.



7E Constructing Angle Bisectors (Exploration)

Expectations

- construct related lines[, and classify triangles, quadrilaterals, and prisms]
- construct angle bisectors [and perpendicular bisectors], using a variety of tools and strategies, and represent equal angles [and equal lengths] using mathematical notation

Answers

C. The line of symmetry divides the angle exactly in half.

H. The angles created by the bisector are equal.

1. Yes. For example, each method divides any angle in half.

2. For example, either divides an angle exactly in half.

3. The measure of each angle you create is half the measure of the angle you bisect. For example, it makes sense because you are creating a line of symmetry.

7F Constructing Perpendicular Bisectors (Exploration)

Expectations

- construct related lines[, and classify triangles, quadrilaterals, and prisms]
- construct [angle bisectors and] perpendicular bisectors, using a variety of tools and strategies, and represent equal angles and equal lengths using mathematical notation

Answers

B. Each angle measure will be 90°.

C. For example, a line of symmetry divides the segment exactly in half.

1. For example, it is necessary for the arc drawn with the compass point on point S to intersect the arc drawn with the compass point on point T so the compass must reach more than half the length of segment ST.

2. For example, each divides the segment into two equal parts.

3. For example, a perpendicular bisector divides an angle in half. When the measure of the angle divided is 180°, half the angle measure is 90°.

4. They show that a perpendicular bisector divides a segment into two equal parts and creates two right angles.

7G Constructing Perpendicular Lines (Exploration)

Expectations

- construct related lines[, and classify triangles, quadrilaterals, and prisms]
- construct related lines (i.e., [parallel;] perpendicular[; intersecting at 30°, 45°, and 60°]), using angle properties and a variety of tools and strategies

Answers

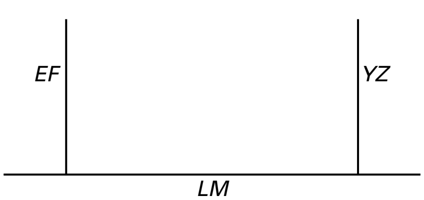
B. Angles will be 90°.

C. They cross or meet the line at 90°.

E. 90°

H. 90°

1. No. For example, line EF does not cross or meet line YZ at right angles.



2. Perpendicular lines divide a straight angle of 180° in half creating angles of 90°.

7H Constructing Parallel Lines (Exploration)

Expectations

- construct related lines[, and classify triangles, quadrilaterals, and prisms]
- construct related lines (i.e., parallel[; perpendicular; intersecting at 30°, 45°, and 60°]), using angle properties and a variety of tools and strategies

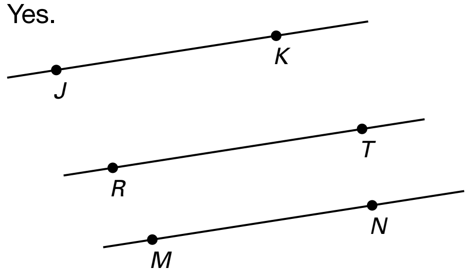
Answers

A. The shortest distance between the lines is always the same.

B. No, they are always the same distance apart.

E. For example, they are the same distance apart. They look as though they will never meet.

1. For example, when a transparent mirror is halfway between two parallel lines, one line is an exact reflection of the other line.
2. Yes.



3. For example, parallel lines never meet or intersect. They are always the same distance apart.

7I Constructing Intersecting Lines (Exploration)

Expectations

- construct related lines[, and classify triangles, quadrilaterals, and prisms]
- construct related lines (i.e., [parallel; perpendicular;] intersecting at 30°, 45°, and 60°), using angle properties and a variety of tools and strategies

Answers

B. For example, the sum of all the angles is 360°. There are two angles of 30° and two angles of 150° on opposite sides of the lines.

C. For example, the sum of all the angles is 360°. There are two angles of 60° and two angles of 120° on opposite sides of the lines.

D. For example, the sum of all the angles is 360°. There are two angles of 45° and two angles of 135° on opposite sides of the lines. The angle measures total 180° on one side of a line, or 360° for all the angle measures.

E. 30°. For example, half the angle measure of 60° is 30° and an angle bisector divides an angle in half.

F. 30°, 60°, 90°, 90°, 90°

G. 45°, 45°

1. For example, if you bisect an angle of 60°, you create two angles each measuring 30°.
2. For example, if you have an angle of 90° and draw an angle inside it of 30°, the remaining part of the angle measures 60°.
3. For example, if you bisect an angle of 90°, you create two angles, each measuring 45°.
4. No. For example, all lines are rotated in the same way so the relationships among the angles do not change.

7A Sorting Triangles and Quadrilaterals

GOAL Sort and classify triangles and quadrilaterals by geometric properties related to symmetry, angles, and sides.

You will need
- square dot paper
- a ruler
- scissors
- a transparent mirror
- a protractor

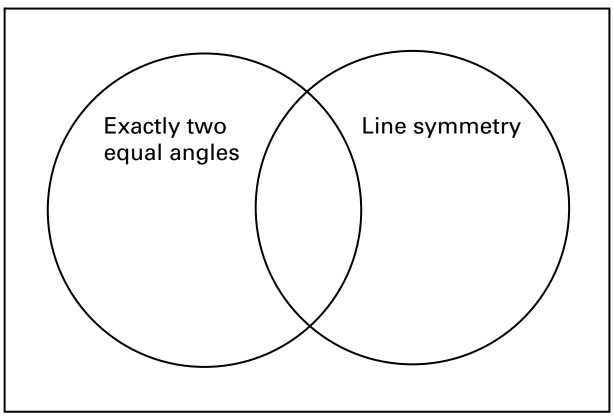
Explore the Math

In a **Venn diagram**, shapes in each circle have a common property. Shapes in the overlap have both properties. Shapes outside the circles have neither property.

? How can you classify triangles and quadrilaterals by sorting?

A. Draw and cut out ten triangles on square dot paper for these conditions.
- at least two right triangles, three acute triangles, and two obtuse triangles
- at least two equilateral triangles, two isosceles triangles, and two scalene triangles

B. Sort your triangles for this Venn diagram. How are the triangles in the overlap different from the triangles that are not in either circle?



C. Make up your own sorting rules and draw a Venn diagram to repeat step B. Include a sorting rule related to sides.

D. Draw and cut out ten quadrilaterals on square dot paper for these conditions.
- at least two parallelograms, two squares, two rectangles, two rhombi, and two quadrilaterals with four sides of unequal lengths

E. Draw a Venn diagram with the circles labelled: *Opposite sides equal and parallel* and *All right angles*. Sort your quadrilaterals. How are the quadrilaterals in the overlap different from the quadrilaterals that are not in either circle?

F. Draw your own Venn diagram to repeat step B. Include symmetry in a sorting rule.

Reflecting

1. Describe how using a Venn diagram can help you classify triangles and quadrilaterals by properties related to symmetry, angles, or sides.

7B Relationships for Congruent Shapes

GOAL Investigate relationships among area, perimeter, corresponding side lengths, and corresponding angle measures of congruent shapes.

Explore the Math

Yuki is comparing the area, perimeter, **corresponding sides**, and **corresponding angles** of congruent shapes.

? How are congruent shapes related?

A. Construct two congruent triangles on a geoboard or on square dot paper.

B. Measure each pair of corresponding angles. What do you notice?

C. Measure each pair of corresponding sides. What do you notice?

D. Calculate the perimeter of each shape. What do you notice?

E. Compare the areas by counting squares and part squares, measuring dimensions and calculating, or placing one shape over the other. What do you notice?

F. Repeat steps A to E for congruent quadrilaterals.

Reflecting

1. Does the orientation of shapes affect which angles are corresponding or which sides are corresponding? Why or why not?

2. a) Suppose you constructed a reflection or rotation image of a polygon. What would you know about corresponding sides and angles of the image and pre-image? Explain why you would know.

b) Suppose you constructed a translation image of a polygon. What would you know about the perimeter and the area of the image and pre-image?

3. Is each statement true? If so, explain why. If not, give an example.

a) Shapes with the same area must be congruent.

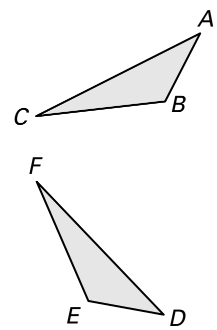
b) Congruent shapes must have the same perimeter.

c) Shapes that have the same perimeter must have corresponding sides of equal length.

You will need
- a geoboard and elastic bands, or centimetre dot paper
- a ruler
- a protractor

corresponding angles

angles that are in the same positions in different shapes



$\triangle ABC$ is congruent to $\triangle DEF$.

Corresponding angles are $\angle A$ and $\angle D$, $\angle B$ and $\angle E$, and $\angle C$ and $\angle F$.

corresponding sides

sides that are in the same positions in different shapes

In $\triangle ABC$ and $\triangle DEF$, corresponding sides are AB and DE, BC and EF, CA and FD.

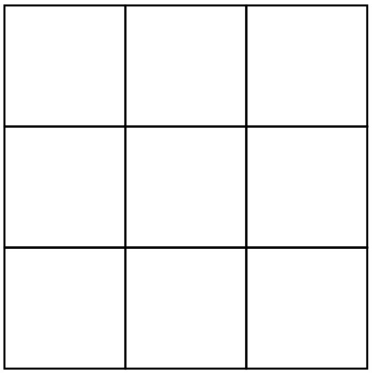
Dilatations with Pattern Blocks

GOAL Identify and describe dilatations created using concrete materials.

You will need
- pattern blocks

Explore the Math

Simon arranged nine square pattern blocks to create this design.



dilatation
a transformation that can enlarge or reduce the size of a figure, but does not change its shape

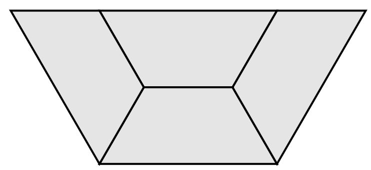
? How can you use pattern blocks to create dilatations of designs?

A. Use four pattern blocks to create a design that is similar to Simon's design, but is a reduction of Simon's design. Sketch your design.

B. Use one pattern block to create a reduction of Simon's design. Sketch your design.

C. How do you know that your designs from steps A and B are similar?

D. Tynessa made this design with trapezoid pattern blocks. Is it similar to a trapezoid pattern block? How do you know? Is it a reduction or an enlargement of a trapezoid pattern block? How do you know?



E. Choose another pattern block that you can use to create a larger shape that is similar to the pattern block. Sketch your design. Are the designs congruent? How do you know?

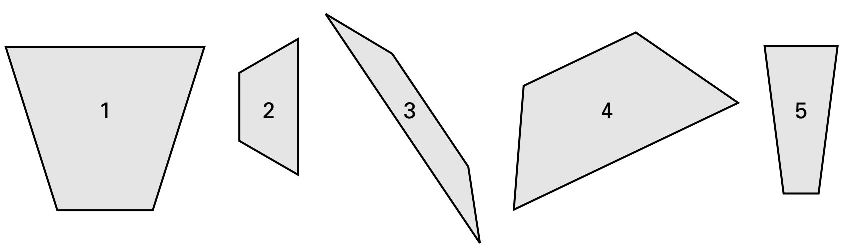
Reflecting

1. Jody created an enlargement of a pattern block design. Paul created a reduction of the same design.

a) Is Jody's design similar to Paul's? How do you know?

b) Is Paul's design an enlargement or a reduction of Jody's? How do you know?

2. Which trapezoids cannot be dilatations of a trapezoid pattern block? Explain.



Dilatations (page 1)

GOAL Explore the properties of dilatation images of 2-D shapes.

You will need
- centimetre grid paper
- a ruler

Learn about the Math

Ryan is making triangular signs to advertise his school play. He wants all the signs to be the same shape, but he wants them to be different sizes so that he can post them in large and small spaces.

? How can Ryan create similar signs?

Example 1: Constructing a dilatation that is an enlargement

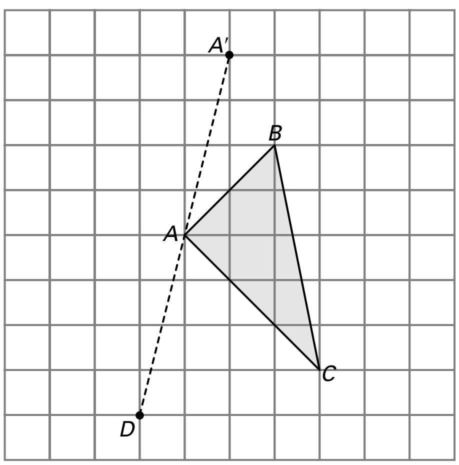
Draw a triangle and a **dilatation centre**.

Construct a dilatation image with a **scale factor** of 2.

dilatation centre
a fixed point from which a segment joins each vertex of a shape to the corresponding vertex of a dilatation image

scale factor
a number by which you multiply the distance between a dilatation centre and a point to get the distance from the dilatation centre to the image point

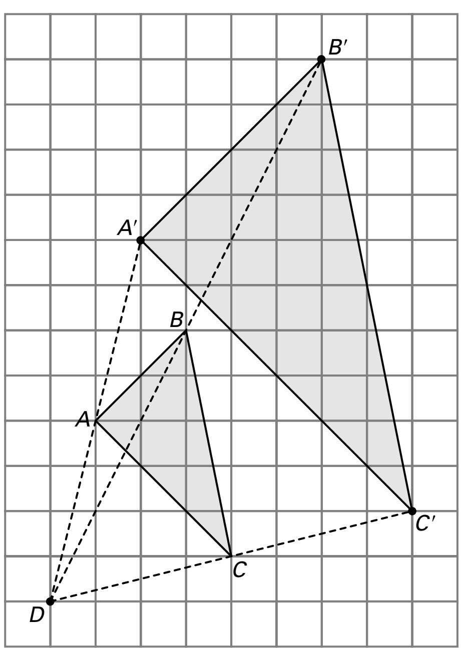
Ryan's Solution



I drew △ABC on grid paper.

I drew point D on a grid mark of the paper for the dilatation centre.

I joined the dilatation centre D with point A. I extended segment DA to mark point A′ so that segment DA′ is twice as long as segment DA.



I used the same method to mark the dilatation image of points B and C. I joined the image points to form △A′B′C′.

Dilatations (page 2)

Reflecting

1. The scale factor for Ryan's dilatation is greater than 1.
   a) Is the image similar to the pre-image?
   b) Is the image an enlargement or reduction of the pre-image?
   c) Is the image closer to, or farther from, the dilatation centre?
   d) Do you think the answers parts (a) to (c) are true for any scale factor greater than 1? Why or why not?

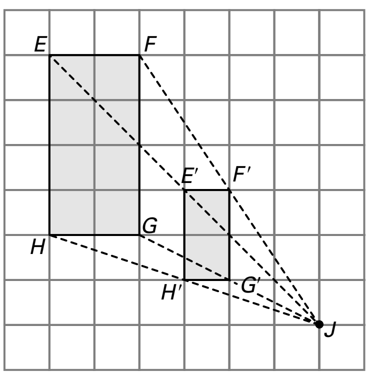
2. What properties of a pre-image and its image are the same after a dilatation? What properties are different? Include orientation.

Work with the Math

Example 2: Describing a dilatation image

Describe the transformation of rectangle EFGH. Include the scale factor.

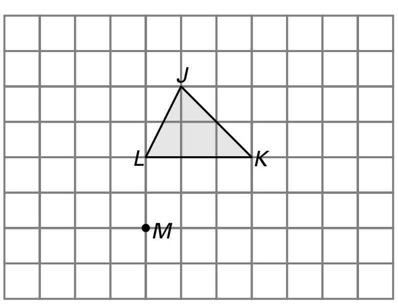
Fawn's Solution



Rectangle E′F′G′H′ is a dilatation image of rectangle EFGH. The dilatation centre is point J. Point E′ is $\frac{1}{2}$ the distance from dilatation centre J as point E. Each of the points F′, G′, and H′ is $\frac{1}{2}$ the distance from dilatation centre J as points F, G, and H. So the scale factor is $\frac{1}{2}$. Rectangle E′F′G′H′ is similar to rectangle EFGH, and it is a reduction of rectangle EFGH.

Ⓐ Checking

3. Dilatate ΔJKL with dilatation centre M and the scale factor of 3. Show the vertices joined to the dilatation centre. Describe the transformation.



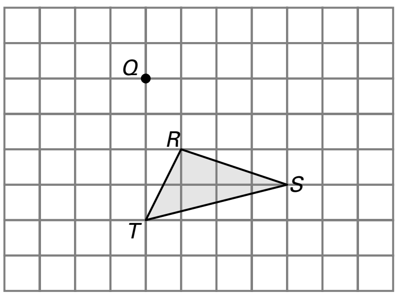
Dilatations (page 3)

B Practising

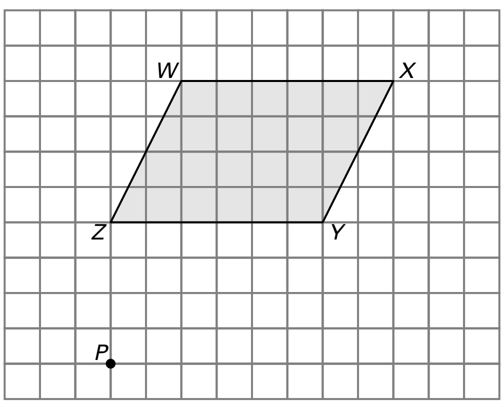
4. Which scale factors for a dilatation result in a reduction? How do you know?

A. 7 **B.** $\frac{1}{4}$ **C.** $\frac{1}{2}$ **D.** 6 **E.** $\frac{1}{5}$

5. Dilatate ΔRST with dilatation centre Q and a scale factor of 2. Show the vertices joined to the dilatation centre. Describe the transformation.



6. Dilatate parallelogram WXYZ with dilatation centre P and a scale factor of $\frac{1}{2}$. Show the vertices joined to the dilatation centre.



7. Use dilatations to create a design. Describe your design.

C Extending

8. When the scale factor is 1, is the image congruent to the pre-image? Is the image in the same position as the pre-image? What do you notice about the image of each point? Include a diagram with your answer.

7E Constructing Angle Bisectors

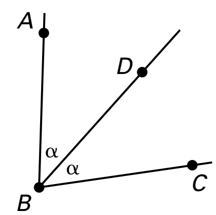
GOAL Investigate methods of constructing an angle bisector.

You will need
- a ruler
- paper for folding
- a protractor
- a transparent mirror
- a compass

Explore the Math

Sandra is exploring methods of constructing an **angle bisector**.

angle bisector
a line that divides an angle into two congruent angles



BD is the angle bisector of ∠ABC. Equal angles are marked with the same symbol such as α, β, or χ.

? How can you construct an angle bisector?

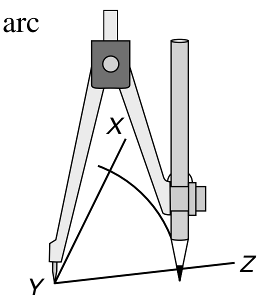
A. Draw ∠DEF. Fold the paper so that one arm of the angle is exactly over the other arm. Unfold the paper. Draw a line along the fold.

B. Measure ∠DEF and each angle created by the angle bisector. Mark the equal angles.

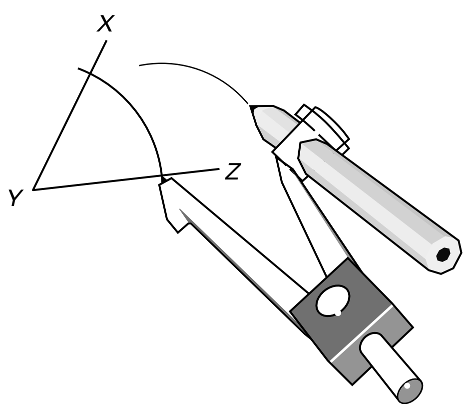
C. Draw ∠JKL. Use a transparent mirror to construct a line of symmetry for ∠JKL. Why does the line of symmetry bisect ∠JKL?

D. Measure ∠JKL and each angle created by the angle bisector. Mark the equal angles.

E. Draw ∠XYZ. With the compass point on point Y, draw an arc through both arms of ∠XYZ.

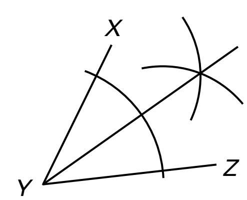


F. With the compass point where the arc crosses YZ, draw an arc between the angle arms.



G. With the compass point where the arc from step E crosses YX, draw an arc to intersect the arc from step F. Join the point of intersection and point Y.

H. Predict which angles are equal. Check your prediction. Mark the equal angles.



Reflecting

1. Can you bisect an acute angle, right angle, or obtuse angle using each method: paper folding, a transparent mirror, and a compass and ruler? Why or why not?

2. How is an angle bisector the same as a line of symmetry?

3. When you bisect an angle, what is the relationship between the measure of each angle you create and the angle you bisect? Why does this make sense?

7F Constructing Perpendicular Bisectors

GOAL Investigate methods of constructing a perpendicular bisector of a segment.

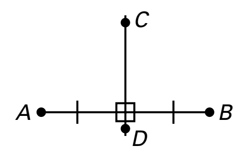
You will need
- a ruler
- paper for folding
- a protractor
- a transparent mirror
- a compass

Explore the Math

Chang is exploring methods of constructing a perpendicular bisector.

? How can you construct a perpendicular bisector?

perpendicular bisector
a line that divides a segment into two congruent segments and meets or crosses the segment at right angles



CD is the perpendicular bisector of AB. Equal segments are marked with the same number of tick marks. Right angles are marked as square corners.

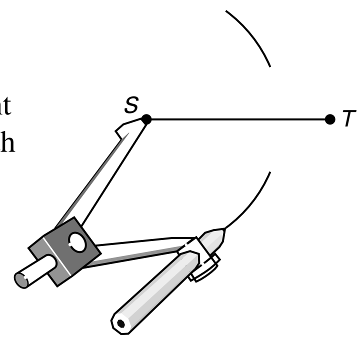
A. Draw segment EF. Fold the paper so that half the segment is exactly over the other half. Unfold the paper. Draw a line along the fold.

B. Measure segment EF and each segment created. Mark the equal lengths. Predict the measure of each angle created. Check your prediction. Mark the right angles.

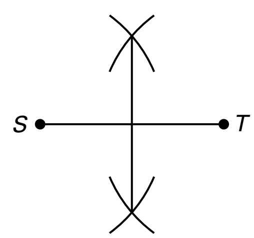
C. Draw segment LM. Use a transparent mirror to construct the line of symmetry. Why is the line of symmetry the perpendicular bisector of segment LM?

D. Mark the equal lengths and right angles.

E. Draw segment ST. Adjust a compass so that the distance between the pencil tip and compass point is greater than half the length of segment ST. With the compass point on point S, draw arcs above and below segment ST.



F. Keep the distance between the pencil tip and compass point the same as in step E. With the compass point on point T, draw arcs above and below segment ST to intersect with the other arcs.



G. Join the points where the arcs intersect. Mark the equal lengths and right angles.

Reflecting

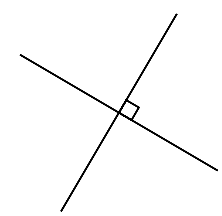
1. In step E, why is it necessary to make the distance between the pencil tip and compass point greater than half the length of segment ST?
2. How is a perpendicular bisector of a segment the same as a line of symmetry?
3. The measure of a straight angle along a segment is 180º. Why is each angle formed by a perpendicular bisector a right angle?
4. How do your constructions show the meaning of a perpendicular bisector?

7G Constructing Perpendicular Lines

GOAL Investigate methods of constructing perpendicular lines, using angle properties.

You will need
- a ruler
- paper for folding
- a protractor
- a transparent mirror
- a compass

perpendicular lines
lines that meet or cross at right angles



Right angles are marked with squares.

Explore the Math

James is exploring methods of constructing **perpendicular lines**.

? How can you construct perpendicular lines?

A. Draw line AB. Fold the paper so that part of the line is exactly over another part of it. Unfold the paper. Draw a line along the fold.

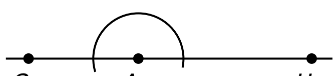
B. Predict the measure of each angle. Check your prediction. Mark the right angles.

C. Draw line CD. Use a protractor to draw an angle of 90º at point C. How do you know the lines are perpendicular? Mark the right angles.

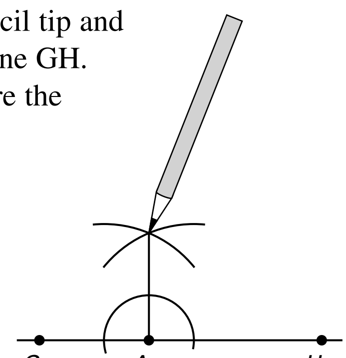
D. Draw line WX. Use a transparent mirror to reflect part of line WX exactly on another part of the line. Draw a line along the transparent mirror to intersect line WX.

E. Measure the angles created. Mark the right angles.

F. Draw line GH. Mark point A on the line. With a compass point at point A, construct an arc that crosses line GH twice.



G. Adjust the compass so that the distance between the pencil tip and compass point is the same to construct two arcs above line GH. Draw one arc with the compass point at each place where the arc from step F crosses line GH.



H. Construct a line joining point A and the point of intersection. Predict the measure of each angle. Check your prediction. Mark the equal angles.

Reflecting

1. Ravi constructed line EF perpendicular to line LM. Then he constructed line YZ perpendicular to line LM. Is line YZ perpendicular to line EF? Use a diagram to explain your answer.

2. The measure of a straight angle along a segment is 180º. Why is each angle formed by perpendicular lines a right angle?

7H Constructing Parallel Lines

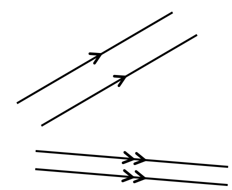
GOAL Investigate methods of constructing parallel lines, using angle properties.

You will need
- a ruler
- paper for folding
- a compass

Explore the Math

Bonnie is exploring methods of constructing **parallel lines**.

parallel lines
lines that never meet or cross



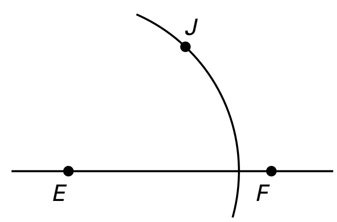
Parallel lines are marked with the same number of small arrows.

? How can you construct parallel lines?

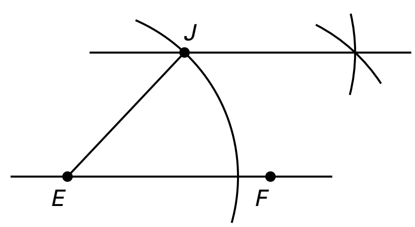
A. Fold a sheet of paper in half so that the straight edges of the paper are together exactly. Fold the paper in half again so that the folded edge and a straight edge of the paper are together exactly. Unfold the paper. Draw a line along each fold. Mark the parallel lines. Measure the shortest distance between the lines at different places. What do you notice?

B. Draw a line along each edge of a ruler. Mark the parallel lines. Do think the lines would meet if you extended them? Why or why not?

C. Draw line EF. Mark point J above the line. Adjust the compass so that the distance between the pencil tip and compass point equals the distance between points E and J. With a compass point on point E, draw an arc through point J that crosses line EF.



D. Keep the distance between the pencil tip and compass point the same as in step C. With the compass point on point J, draw an arc. Then with the compass point on the point where the arc from step C crosses the line, construct another arc so that these arcs intersect.



E. Construct a line through point J and the point where the arcs from step D intersect. Mark the parallel lines. What do you notice about the lines?

Reflecting

1. Use a transparent mirror to check whether the lines you constructed are parallel. Explain your method.
2. Miguel constructed line JK parallel to line RT. Then she constructed line MN parallel to line RT. Is line MN parallel to line JK? Use a diagram to explain your answer.
3. What do your constructions show about parallel lines?

Constructing Intersecting Lines

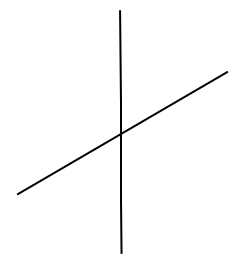
GOAL Investigate methods of constructing lines that intersect at 30°, 45°, and 60°, using angle properties.

You will need
- a ruler
- paper for folding
- a protractor
- a transparent mirror
- a compass

Explore the Math

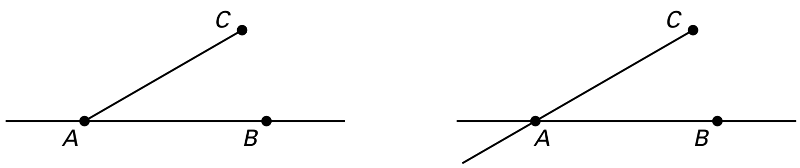
Colin is using angle properties to explore methods of constructing **intersecting lines**.

intersecting lines
lines that meet or cross



? How can you construct lines that intersect at 30°, 45°, or 60°?

A. Draw line AB. Use a protractor to construct an angle with the measure 30º at point A. Mark point C on the angle. Extend CA to cross line AB.



B. Measure the other angles at the point of intersection. Record the measure on each angle. What do you notice about the angle measures?

C. Use a method similar to step A to construct lines that intersect at 60º. Measure the angles. What do you notice?

D. Use a method similar to step A to construct lines that intersect at 45º. Measure the angles. What do you notice?

E. Use a protractor to construct lines that intersect at 60º. Bisect an angle that measures 60°. Measure the angles formed by the angle bisector. Why does this make sense?

F. Use a method of your choice to construct perpendicular lines. Use a protractor to construct an angle of 30° at the point of intersection. What are all the angle measures?

G. Use a method of your choice to construct perpendicular lines. Suppose you bisected one of the angles formed by the perpendicular lines. Predict the angle measure of each angle formed by the angle bisector. Check your prediction.

Reflecting

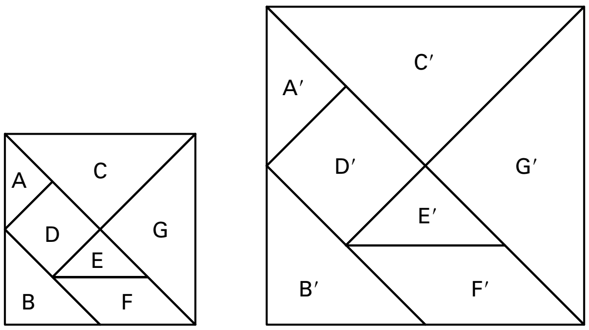
1. Why can you use the method in step E to construct lines that intersect at 30º?
2. Why can you use the method in step F to construct lines that intersect at 60º?
3. Why can you use the method in step G to construct lines that intersect at 45º?
4. When you turn the paper to change the orientation of intersecting lines, do the angle measures change? Why or why not?

Name: ______________________________ Date: ____________________

Chapter 7 Supplemental Task

Constructing and Analyzing Tangrams

For a math project, Simon and Ravi made two tangrams, one twice the size of the other. They measured the tangram pieces to find relationships between them.



? How can you construct tangrams of different sizes and relate the pieces?

A. Draw a tangram. There are three requirements.
- You may use only a ruler and a compass and plain paper.
- To draw the tangram, you must construct at least one parallel line, one perpendicular line, one angle bisector, and one perpendicular bisector.
- The sides of the tangram must be either 7 cm or 14 cm.

B. On a piece of grid paper, construct a dilatation image of any one piece of the tangram drawn in part A. Use a scale factor of 2.

C. Use a tangram piece to create a larger piece that is similar to that piece. Sketch your design.

D. Write two sorting rules for tangram pieces that use properties related to symmetry, angles, and sides. Use these rules to sort the 7 tangram pieces in a Venn diagram.

E. Describe which of the 14 pieces in Simon and Ravi's two tangrams are similar and which are congruent. Explain how you know.

Task Checklist

- Did you use the correct drawing tools?
- Did you demonstrate each type of construction specified?
- Is your tangram the correct size?
- Is your dilatation image twice the size of the pre-image?
- Does your sorting rule use properties related to symmetry, angles, and sides?
- Did you describe which pieces of the tangrams are congruent and which are similar?

Chapter 7 Supplemental Task

Expectations

- construct related lines, and classify triangles, quadrilaterals [, and prisms]
- develop an understanding of similarity, and distinguish similarity and congruence
- [describe location in the four quadrants of a coordinate system,] dilatate two-dimensional shapes, and apply transformations to [create and] analyse designs
- construct related lines (i.e., parallel; perpendicular; intersecting at [30°,] 45°[, and 60°]), using angle properties and a variety of tools and strategies
- sort and classify triangles and quadrilaterals by geometric properties related to symmetry, angles, and sides, through investigation using a variety of tools
- construct angle bisectors and perpendicular bisectors, using a variety of tools and strategies, and represent equal angles and equal lengths using mathematical notation
- determine, through investigation using a variety of tools, relationships among area, perimeter, corresponding side lengths, and corresponding angles of congruent shapes
- demonstrate an understanding that enlarging or reducing two-dimensional shapes creates similar shapes
- distinguish between and compare similar shapes and congruent shapes, using a variety of tools and strategies
- identify, perform, and describe dilatations (i.e., enlargements and reductions), through investigation using a variety of tools
- create and analyse designs involving translations, reflections, dilatations, and/or simple rotations of two-dimensional shapes, using a variety of tools and strategies

Use this task as a performance assessment, to give you a sense of students' understanding of congruence and similarity of 2-D shapes, their ability to construct parallel and perpendicular lines and angle and perpendicular bisectors, and their ability to dilatate 2-D shapes.

Preparation and Planning

Pacing	**5 min** Introducing the Chapter Supplemental Task **40 min** Using the Chapter Supplemental Task
Materials	• a ruler • a compass • a protractor • a transparent mirror • 1 cm Grid Paper, Masters Booklet p. 34
Enabling Activities	Sorting Triangles and Quadrilaterals (See Lesson 7A.) Relationships for Congruent Shapes (See Lesson 7B.) Dilatations with Pattern Blocks (See Lesson 7C.) Dilatations (See Lesson 7D.) Constructing Angle Bisectors (See Lesson 7E.) Constructing Perpendicular Bisectors (See Lesson 7F.) Constructing Perpendicular Lines (See Lesson 7G.) Constructing Parallel Lines (See Lesson 7H.) Constructing Intersecting Lines (See Lesson 7I.)
Nelson Web Site	• Visit **www.mathK8.nelson.com** and follow the links to *Nelson Mathematics 7*, Chapter 7, for assessment support notes.

Introducing the Chapter Supplemental Task

(Whole Class) ▸ about 5 min

Have students look at the diagrams of the tangrams and identify the parallel and perpendicular lines and angle and perpendicular bisectors in a tangram, and suggest strategies they could use to construct the various parts of a tangram.

Using the Chapter Supplemental Task

(Pairs) ▸ about 40 min

Read through the task together. Point out the requirements for the construction of the tangram. Ask, "How will you determine which strategy will best suit each part of the tangram and yet still demonstrate each of the four types of construction specified?"

Have students work in pairs. For prompts A to C, students will need a ruler and compass. For prompt B, students will also need a sheet of centimetre grid paper. Remind students to make a plan and to show their work, including the pencil marks made with the compass, clearly on their drawings. For prompts D and E, a protractor and a transparent ruler are required.

Remind students to use the task checklist to help them produce a good solution. While students are working, observe and/or interview individuals to see how they are interpreting and carrying out the task.

Assessing Students' Work

Use the Assessment of Learning chart as a guide for assessing students' work.

Adapting the Task

You can adapt the task to suit the needs of your students. For example:

- Provide a master of the outer square of the tangram as a starting point for construction of the tangram.
- Permit students to refer to Lessons D to I for construction steps.
- Some students may need to use concrete tangram pieces to identify geometric relationships and to create and analyse designs involving dilatations.
- Challenge stronger students to sketch a tangram using only a ruler.

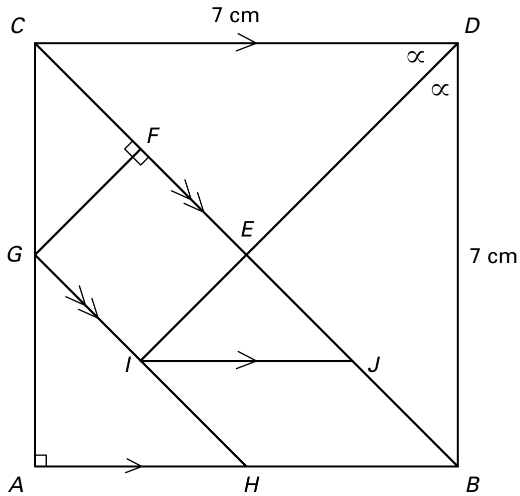
Assessment of Learning—What to Look for in Student Work…

Assessment Strategy: Interview/Observation and Product Marking
Problem Solving/Thinking

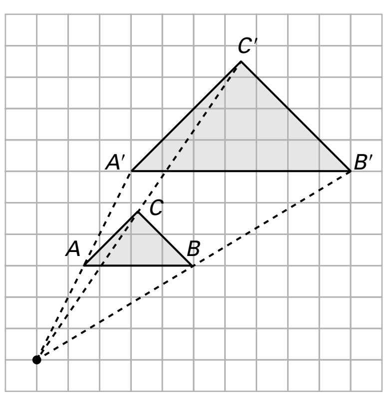
Level of Perfomance	**1**	**2**	**3**	**4**
Problem Solving/Thinking Prompt A Think: Understand the Problem	• shows **limited** understanding of the problem	• shows **some** understanding of the problem	• shows **complete** understanding of the problem	• shows **thorough** understanding of the problem
Prompt A Do: Carry Out the Plan	• uses a strategy and **attempts** to solve problem but **does not arrive at an answer**	• carries out the plan **to some extent**, using a strategy, and develops a **partial and/or incorrect solution**	• carries out the plan **effectively** by using an **appropriate** strategy and **solving the problem**	• shows **flexibility** and **insight** when carrying out the plan by **trying** and **adapting**, when necessary, **one or more** strategies to **solve the problem**
Prompts A & B Do: Carry Out the Plan	• use of procedures includes **major errors and/or omissions**	• use of procedures includes **several errors and/or omissions**	• use of procedures is mostly correct, but there may be a **few minor errors and/or omissions**	• use of procedures includes **almost no errors or omissions**
Knowledge & Understanding Prompts A & B Depth of Knowledge	• demonstrates a **limited** or **inaccurate** knowledge of the specific terms and/or procedural skills that have been taught	• demonstrates **some** knowledge of the specific terms and/or procedural skills that have been taught	• demonstrates **considerable** knowledge of the specific terms and/or procedural skills that have been taught	• demonstrates **thorough** knowledge of the specific terms and/or procedural skills that have been taught
Knowledge & Understanding Prompts C, D, & E Depth of Understanding	• demonstrates a **limited** or **inaccurate** understanding of concepts	• demonstrates **some** understanding of concepts	• demonstrates **considerable** understanding of concepts	• demonstrates **thorough** understanding of concepts
Application of Learning Prompts A, C, & E Transferring knowledge and skills to new contexts	• demonstrates **limited** ability to transfer mathematical knowledge and skills to new contexts	• demonstrates **some** ability to transfer mathematical knowledge and skills to new contexts	• demonstrates **considerable** ability to transfer mathematical knowledge and skills to new contexts	• demonstrates **sophisticated** ability to transfer mathematical knowledge and skills to new contexts
Communication Prompts A, B, & C Use of mathematical representations (graphs, charts, diagrams)	• uses representations that exhibit **limited clarity** and **accuracy**, and are **ineffective** in communicating	• uses representations that exhibit **some clarity** and **accuracy**	• uses representations that are **clear** and **accurately** communicate information	• uses representations that are **clear**, **precise**, and **effective** in communicating
Prompts A–E Use of mathematical conventions (units, symbols, labels)	• **few** conventions are used correctly	• **some** conventions are used correctly	• **most** conventions are used correctly	• **almost all** conventions are used correctly

Answers

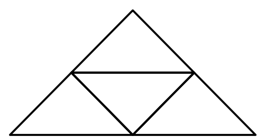
A. For example, draw a 7 cm line segment *AB*. Draw a perpendicular 7 cm line segment *AC*. Draw a 7 cm line segment *CD* parallel to *AB*. Connect *D* and *B*. Draw the diagonal *CB*. Construct the angle bisector *DE* bisecting ∠*CDB*. Construct *FG*, the perpendicular bisector of *CE*. Construct *GH* parallel to *CB*. Extend *DE* to *DI*. Construct *IJ* parallel to *AB*.



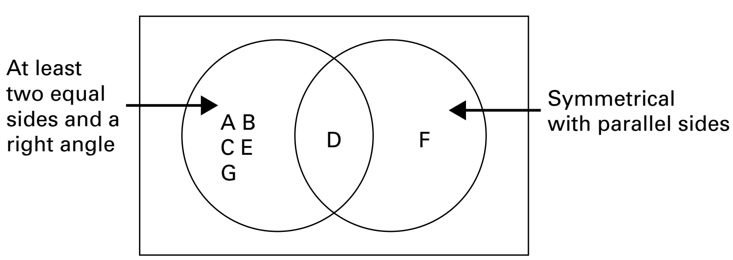
B. For example,



C. For example,



D. For example, "at least two equal sides and a right angle" and "symmetrical with parallel sides."



E. For example, A is similar to A′, B is similar to B′, and so on for all the pieces, because the tangram shapes have not changed, only the size of the tangram has changed. All of the triangles are similar, because they are all right icosceles triangles. C and G are congruent, and C′ and G′ are congruent, because you can see from the design that all the corresponding side lengths and angles are equal. A and E are congruent, and A′ and E′ are congruent, because I measured and their corresponding angles and sides are equal.

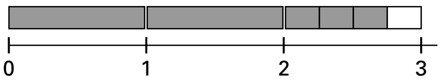
Chapter 9 Expectations and Answers

9A Comparing Fractions and Decimals (Exploration)

Expectations

- represent, compare, and order numbers [,including integers]
- represent, compare, and order decimals to hundredths and fractions using a variety of tools

Answers

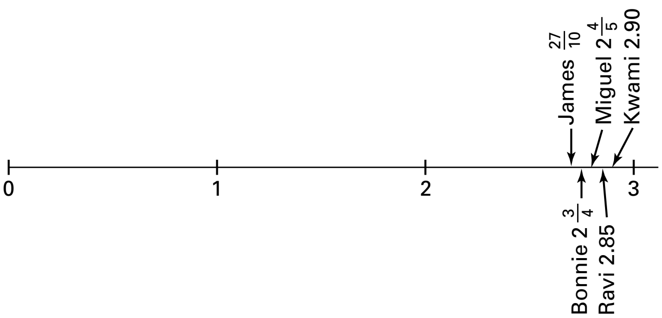
A.–B. 

C. 

D. I can use fraction strips for tenths because 2.90 equals 2.9 or 2 and 9 tenths.

E. I marked the point for Ravi between $2\frac{4}{5}$ and 2.90 because $2\frac{4}{5}$ equals 2.8 and 2.85 is between 2.8 and 2.9.

F. I marked the point for James to the left of $2\frac{4}{5}$ because $\frac{27}{10}$ equals $2\frac{7}{10}$ or 2.7 which is less than 2.8.



G. Kwami, Ravi, Miguel, Bonnie, James

H. Kwami

1. For example, I could use a calculator to convert the fraction part of each mixed number to a decimal such as divide 3 by 4 and add 2 to change $2\frac{3}{4}$ to a decimal, and divide 27 by 10 to change $\frac{27}{10}$ to a decimal. Then I could compare the order of the decimals.

2. Jody memorized more songs than the other students because 3 songs and a part of another is greater than 2.90 which is what Kwami memorized.

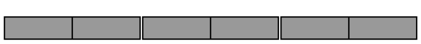
9B Dividing a Whole Number by a Fraction (Exploration)

Expectations

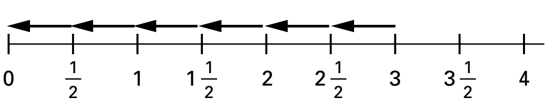
- [demonstrate an understanding of addition and subtraction of fractions and integers, and] apply a variety of computational strategies to solve problems involving whole numbers [and decimal numbers]
- divide whole numbers by simple fractions [and by decimal numbers to hundredths], using concrete materials

Answers

A. 2; each key chain takes $\frac{1}{2}$ of a sheet of plastic.

B. 3 

C. 3 divided by $\frac{1}{2}$ results in six halves which show the number of key chains Yuki can make is six.

D. 

E. At 3; Yuki has 3 sheets of plastic. The arrows go to the left because each key chain takes $\frac{1}{2}$ of a sheet so you are taking away $\frac{1}{2}$ of the sheet of plastic every time you make a key chain. The last arrow ends at 0 because that shows using all the sheets of plastic. There are 6 arrows.

F. It shows how 3 sheets of plastic are being divided into 6 sections, each representing $\frac{1}{2}$.

1. For example, the model shows that when you have 3 fraction strips and you divide each strip in half, you have a total of 6 key chains.

2. For example, I subtracted $\frac{1}{2}$ six times.

3. a) $2 \div \frac{1}{5} = 10$ for 10 key chains

b) $6 \div \frac{2}{3} = 9$ for 9 key chains

9C Adding and Subtracting Fractions Mentally (Exploration)

Expectations

- demonstrate an understanding of addition and subtraction of fractions [and integers, and apply a variety of computational strategies to solve problems involving whole numbers and decimal numbers]
- use a variety of mental strategies to solve problems involving the addition and subtraction of fractions [and decimals]

Answers

A. $\frac{2}{4}$; $\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$ which equals $\frac{1}{2}$.

B. $2\frac{1}{4}$; I thought of $\frac{1}{2}$ as $\frac{2}{4}$ and added $1\frac{3}{4} + \frac{1}{4} + \frac{1}{4} = 1 + \frac{4}{4} + \frac{1}{4}$ which gives $1 + 1 + \frac{1}{4}$ or $2\frac{1}{4}$.

C. $\frac{1}{4}$; Colin started with $2\frac{1}{2}$ cups and used $2\frac{1}{4}$. $2\frac{1}{2} - 2\frac{1}{4} = 2\frac{2}{4} - 2\frac{1}{4}$ which gives $\frac{1}{4}$.

1. a) For example, subtract $\frac{1}{2}$ from $2\frac{1}{2}$ which leaves 2. Subtract $2 - 1\frac{3}{4}$ which leaves $\frac{1}{4}$.

b) For example, I like the method in steps A to C because I can think of the total needed and then the difference.
For example, I like Indira's method because it has fewer steps.

2. For example, it is easier to calculate $\frac{4}{5} - \frac{3}{5} = \frac{1}{5}$ mentally because the denominators are the same.

3. a) Chang grouped fractions together that had the sum 1 and added the other fraction.

b) Rana grouped fractions so she could subtract to get 1, and then subtract the fraction $\frac{1}{2}$ from 1.

4. a) For example, group $\frac{9}{10} + \frac{1}{10} = 1$, and add $1 + \frac{1}{5} = 1\frac{1}{5}$.

b) For example, think $\frac{5}{6} - \frac{5}{6} = 0$. That leaves $\frac{5}{6}$.

c) For example, think $\frac{5}{8} + \frac{3}{8} = 1$, then add $1 + \frac{1}{4} = 1\frac{1}{4}$.

d) For example, think $\frac{11}{10} - \frac{1}{10} = 1$, then subtract $1 - \frac{1}{2} = \frac{1}{2}$.

9A Comparing Fractions and Decimals

GOAL Use a variety of tools to compare and order fractions and decimals.

You will need
- fraction strips
- a calculator

Explore the Math

Some friends are memorizing new songs on a CD.

Bonnie said, “I memorized 2 songs and $\frac{3}{4}$ of another song.”

Miguel said, “I memorized 2 songs and $\frac{4}{5}$ of another song.”

Kwami said, “I memorized 2.90 songs.”

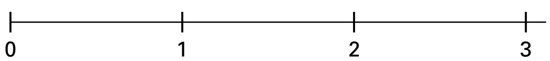
Ravi said, “I memorized 2.85 songs.”

James said, “I memorized $\frac{27}{10}$ songs.”

? Who memorized the most songs?

A. Use fraction strips to represent the amount Bonnie memorized.

B. Use your fraction strips from step A to draw a number line and mark the amount Bonnie memorized.



C. Use fraction strips to represent the amount Miguel memorized. Use your fraction strips to mark this amount on your number line from step B.

D. Use fraction strips to represent the amount Kwami memorized. Why can you use tenth fraction strips? Use your fraction strips to mark the amount Kwami memorized on your number line.

E. Mark the amount Ravi memorized on your number line. Explain your strategy.

F. Repeat step E for James.

G. List the students in order from who memorized the most songs.

H. Who memorized the most songs?

Reflecting

1. Tell how to use a calculator to check the order of the fractions and decimals.

2. Jody memorized three songs and part of another. How do you know whether this is more or fewer songs than the other students?

9B Dividing a Whole Number by a Fraction

GOAL Use models to divide whole numbers by fractions.

You will need
- fraction strips

Explore the Math

Yuki has three sheets of plastic to make key chains. She needs half a sheet for each key chain.

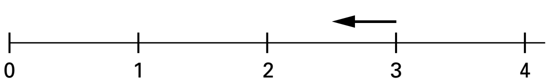
? How many key chains can Yuki make?

A. How many key chains can Yuki make with one sheet of plastic? How do you know?

B. How many fraction strips do you need to represent all of Yuki's sheets of plastic? Use fraction strips to calculate the number of key chains she can make.

C. How do your fraction strips show the number of key chains Yuki can make?

D. Use a number line to show Yuki's sheets of plastic. Draw arrows to calculate the number of key chains she can make.



E. Where does your first arrow start? Why? What direction do your arrows go? Why? Where does your last arrow end? Why? How many arrows did you draw?

F. How does your number line show the number of key chains Yuki can make?

Reflecting

1. Explain how each model shows $3 \div \frac{1}{2} = 6$.

2. Choose your fraction strip or number line model. How does it show $3 - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} = 0$. How does the repeated subtraction show the number of key chains Yuki can make?

3. Use a model to determine how many key chains each can make. Explain your thinking.

a) Simon has two sheets of plastic. He needs $\frac{1}{5}$ of a sheet to make each key chain.

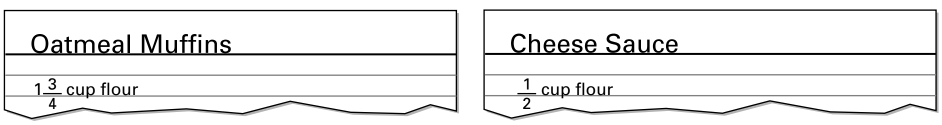
b) Fawn has six sheets of plastic. She needs $\frac{2}{3}$ of a sheet to make each key chain.

9C Adding and Subtracting Fractions Mentally

GOAL Solve problems by adding and subtracting fractions mentally.

Explore the Math

Colin has $2\frac{1}{2}$ cups of flour to make the oatmeal muffins and cheese sauce.



? How much flour will Colin have left?

A. How many quarters of a cup does Colin need for the sauce? How do you know?

B. Using mental strategies, tell how much flour Colin needs altogether. Explain your thinking.

C. Using mental strategies, tell how much flour Colin will have left. Explain your thinking.

Reflecting

1. a) Indira calculated the amount of flour Colin will have left by thinking $2\frac{1}{2} - \frac{1}{2}$, $2 - 1\frac{3}{4}$. Describe her strategy. Explain how to finish her calculations mentally.

b) Do you prefer the strategy in steps A to C or Indira's strategy? Explain your choice.

2. Which is easier to calculate mentally: $\frac{4}{5} - \frac{3}{5}$ or $\frac{4}{5} - \frac{3}{10}$? Why?

3. a) Chang calculated $\frac{1}{5} + \frac{2}{5} + \frac{4}{5}$ by thinking $\frac{1}{5} + \frac{4}{5} = \frac{5}{5}$ or 1, $1 + \frac{2}{5} = 1\frac{2}{5}$. Describe his strategy.

b) Rana calculated $\frac{7}{4} - \frac{1}{2} - \frac{3}{4}$ by thinking $\frac{7}{4} - \frac{3}{4} = \frac{4}{4}$ or 1, $1 - \frac{1}{2} = \frac{2}{2} - \frac{1}{2}$ which equals $\frac{1}{2}$. Describe her strategy.

4. Explain how to calculate each using mental strategies.

a) $\frac{9}{10} + \frac{1}{5} + \frac{1}{10}$ **c)** $\frac{1}{4} + \frac{5}{8} + \frac{3}{8}$

b) $\frac{5}{6} + \frac{5}{6} - \frac{5}{6}$ **d)** $\frac{11}{10} - \frac{1}{2} - \frac{1}{10}$

10A Describing Right Prisms

GOAL Investigate between faces for right prisms, and identify right prisms.

You will need
- 3-D models
- pattern blocks
- a protractor

Explore the Math

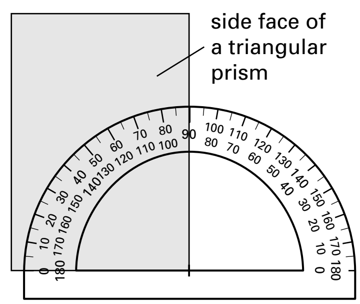
Miguel thinks there is a relationship between angle measures at **vertices** of **right prisms** and which **faces** are perpendicular.

right prism
a prism with two congruent polygons joined by rectangular sides

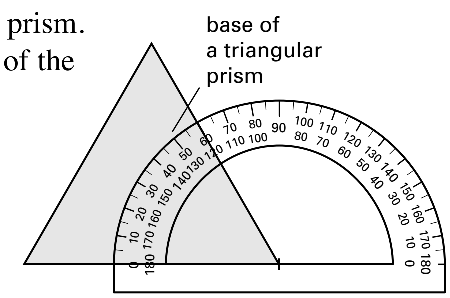
? How do angle measures show which faces of a right prism are perpendicular?

A. How can you identify a right prism by looking at the base? Give two examples.

B. Measure each angle on a side face of a triangular prism. What are the angle measures? What are the angle measures at the vertices of the other side faces? What polygon is each side face? Is each side face perpendicular to the base?

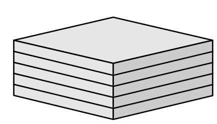


C. Measure each angle on the base of the triangular prism. What are the angle measures? Are the side faces of the prism perpendicular to each other?



D. Repeat steps B and C for a rectangular prism.

E. Repeat steps B and C for three other right prisms. Use 3-D models and right prisms constructed by stacking pattern blocks.



Reflecting

1. a) How do the angle measures on the side faces of a right prism show whether the side faces are rectangles?

b) How do the angle measures on the side faces of a right prism show whether the base is perpendicular to the base?

2. What do your answers in question 1 show about right prisms?

3. What does your investigation show about whether faces of right prisms are perpendicular to each other?

Chapter 11 Expectations and Answers

11A Surface Area of a Right Prism (Guided Activity)

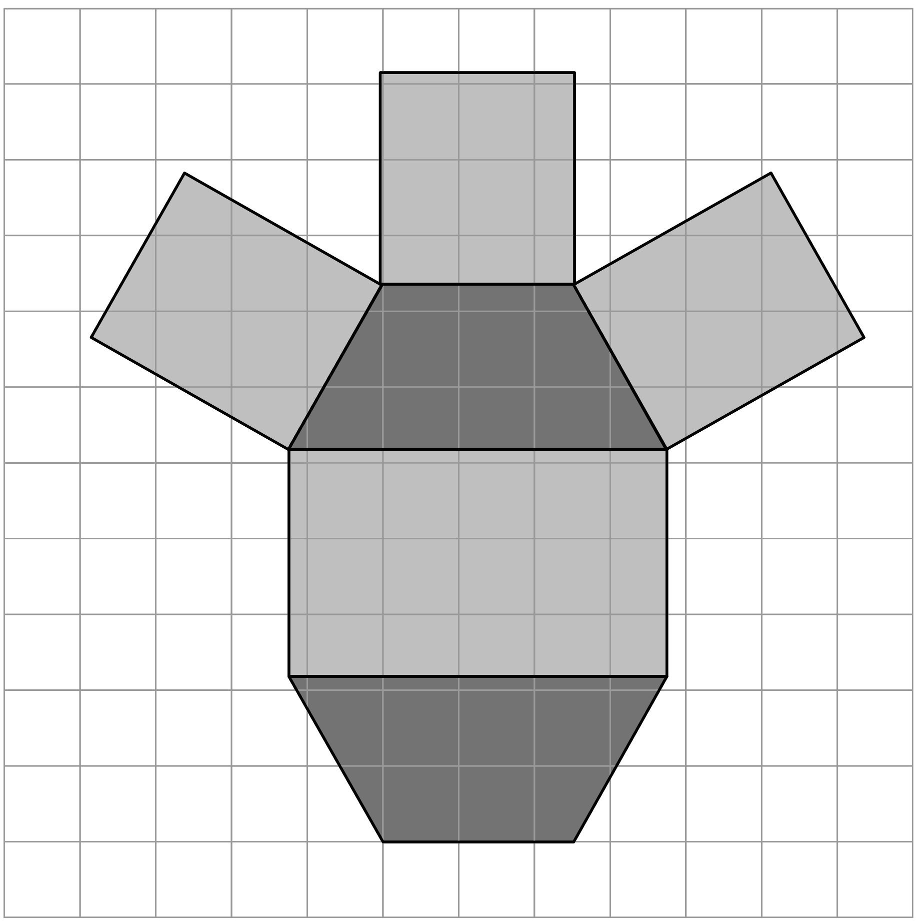
Expectations

- explain the relationship between exponent notation and the measurement of area [and volume]
- solve problems that require conversion between metric units of measure
- determine, through investigation using a variety of tools, the surface area of right prisms
- solve problems that involve the surface area [and volume] of right prisms [and that require conversion between metric measurements of capacity and volume (i.e., millilitres and cubic centimetres)]

Answers

Note: Answers may vary according to the thickness of the pattern blocks or the 3-D models.

B. For example,



For example, about 50 cm^2

C. 7.9 cm^2

D. 1

E. 7.2 cm^2, 14.4 cm^2, 7.2 cm^2, 7.2 cm^2; the two rectangles along the slanted sides of the trapezoid and the rectangle along the shorter parallel side of the trapezoid are congruent.

F. 51.8 cm^2. I added all of the areas of the faces.

G. The areas are close.

1. For example, I looked for part squares that fit together to make whole squares.

2. For example, multiply the number of congruent faces times the area of one of the congruent faces.

3. Sandra can determine the area of each face and then add the areas. Surface area is the sum of the area of all faces.

4. For example, area can be determined by counting squares. Area is length times width. If the unit of each dimension is centimetres, the unit for the area is centimetres times centimetres.
5. **b)** For example, area of each triangle is about 3 cm^2; area of each rectangle is about 18 cm^2
   c) For example, about 60 cm^2
   d) I could calculate the area of the triangle and multiply it by 2, and then find the area of the rectangle, and multiply it by 3. Then I would add the two numbers to get the area.
6. **b)** For example, area of each pentagon is about 4 cm^2; area of each rectangle is about 12 cm^2
   c) For example, about 68 cm^2
   d) For example, the top and bottom are congruent so I only traced one. The five sides are congruent so I only traced one.
7. **b)** For example, area of a base is about 5 cm^2; area of each side rectangle is about 5 cm^2
   c) about 30 cm^2
   d) For example, the top and bottom are congruent so I only traced one. The four sides are congruent so I only traced one.
8. 4.0 m^2
9. For example, about 140 cm^2
10. No. For example, when you double the height of a hexagonal prism, you double only the side faces. You do not double the top and bottom of the prism.

11B Volume of a Right Prism (Guided Activity)

Expectations

- explain the relationship between exponent notation and the measurement of area and volume
- determine the relationships among units and measurable attributes, including the area of a trapezoid and volume of a right prism
- solve problems that require conversion between metric units of measure
- determine, through investigation using a variety of tools and strategies, the relationship between the height, the area of the base, and the volume of right prisms with simple polygonal bases, and generalize to develop the formula (i.e., *volume* = *area of base* × *height*)
- solve problems that involve the surface area and volume of right prisms and that require conversion between metric measurements of capacity and volume (i.e., millilitres and cubic centimetres)

Answers

A. The area of the base times the height, or number of layers which is 1, gives the volume.
B. The area of the base times the height, or number of layers which is 2, gives the volume.
C.–D.

Base of prism	Height of prism	Volume of prism
Area of the base of rhombus pattern block	1 layer	1 rhombus pattern block
Area of the base of rhombus pattern block	2 layers	2 rhombus pattern blocks
Area of the base of rhombus pattern block	3 layers	3 rhombus pattern blocks
Area of the base of rhombus pattern block	4 layers	4 rhombus pattern blocks
Area of the base of rhombus pattern block	5 layers	5 rhombus pattern blocks
Area of the base of rhombus pattern block	6 layers	6 rhombus pattern blocks

E. The height or number of layers would be the same. The relationship among the base, the height, and the volume would be the same. The area of the base would be the area of the base of a hexagon pattern block, and the volume would be the number of hexagon pattern blocks.

1. The volume is the area of the base of a rhombus pattern block times the height of the prism, or the number of layers. The relationship between the area of the rectangular base and the volume is the same for a rectangular prism.

2. Paul can calculate the area of the base of a box and multiply the area of the base times the height.

3. Volume = Area of base × height

4. **a)** For example, volume can be determined by counting cubes. Volume is length times width times height. If the unit of each dimension is centimetres, the unit for the volume is centimetres times centimetres times centimetres.

b) For example, area can be determined by counting squares. Area is length times width. If the unit of each dimension is centimetres, the unit for the area is centimetres times centimetres.

5. **a)** 400 cm^3

b) 2000 cm^3

c) 16 000 cm^3

d) 7456 cm^3

6. **a)** 144 cm^3

b) 245 cm^3

7. **a)** 336 cm^3

b) 1350 cm^3

8. **a)** prism in part a): 336 mL; prism in part b): 1350 mL

b) prism in part a): 0.336 L; prism in part b): 1.350 L

9. 32 400 cm^3

10. 70 cm

11. 45 cm^2

12. **a)** Yes. For example, 6 triangle pattern blocks can be arranged to have the same shape and size as a hexagon pattern block. The volume of the stack of pattern blocks is the same as the total volume of all the pattern blocks.

b) No. For example, when the pattern blocks are stacked, some faces are covered so the total surface area of all the pattern blocks is greater than the surface area of the stack of pattern blocks.

13. **a)** The volume is multiplied by 3. For example, the formula for the volume of a right prism is that the volume of the prism is the area of the base times the height. When the height is multiplied by 3 so is the volume.

b) The volume is halved. For example, the formula for the volume of a right prism is that the volume of the prism is the area of the base times the height. When the area of the base is divided by 2 so is the volume.

c) Yes. The relationship among the volume, area of the base, and the height is the same for any right prism.

11A Surface Area of a Right Prism (page 1)

GOAL Determine the surface area of right prisms and solve surface area problems.

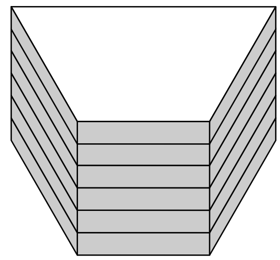
You will need
- pattern blocks
- centimetre paper grid
- a ruler
- a calculator
- 3-D models

Learn about the Math

Sandra is making boxes to hold mystery prizes at a fair. The boxes will be different right prisms. She is going to tape pieces of construction paper together without any overlap.

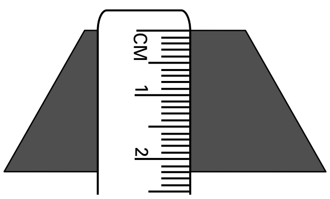
? How can Sandra figure out how much construction paper she needs for a box?

A. Stack six trapezoid pattern blocks to construct a trapezoidal prism.



B. Create a net by tracing each face of the prism on centimetre grid paper. Estimate the area of your net in square centimetres by counting squares and part squares.

Use your trapezoidal prism for steps C to F.



C. Measure the dimensions to the nearest tenth of a centimetre that you need to calculate the area of the base. Use a calculator. What is the area to the nearest tenth of a square centimetre?

D. How many faces are congruent to the base?

E. Look at one of the rectangular faces of your prism. Measure the dimensions to the nearest tenth of a centimetre that you need to calculate the area of the rectangle. Use a calculator. What is the area to the nearest tenth of a square centimetre? Repeat this for the other rectangular faces. Are any of the rectangular faces congruent?

F. What is the sum of the areas of the faces of the prism to the nearest tenth of a square centimetre? How do you know?

G. Compare the surface area in step F with your estimate in step B.

Reflecting

1. Explain your strategy for counting part squares on your net in step B.

2. How could you use congruent faces to help you calculate the surface area in step F?

3. Explain how Sandra can determine the amount of cardboard she needs for any of her boxes. Why is this the same as determining the surface area of a box?

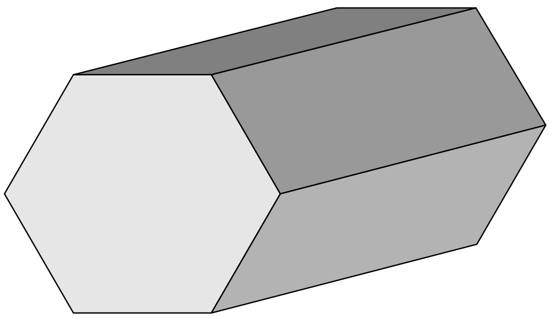
4. Explain why surface area is expressed in square units such as cm^2 with the exponent 2.

Surface Area of a Right Prism (page 2)

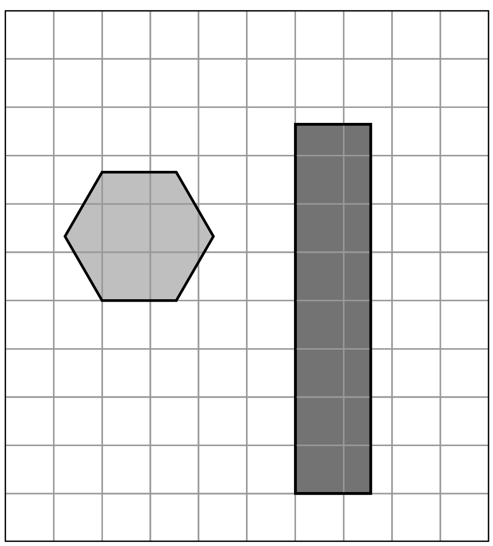
Work with the Math

Example: Tracing faces of a 3-D model

About how many square centimetres is the surface area of this hexagonal prism?



Chang's Solution



I traced the base on centimetre grid paper. To determine the area, I counted the squares and matched part squares to make whole squares. It's about 6 squares, so the area of the base is about 6 cm^2.

I traced a rectangular side of the prism on centimetre grid paper. I counted about 11 squares so the area of the side is about 11 cm^2.

$2 \times 6\ cm^2 = 12\ cm^2$	The top face is congruent to the base, so I multiplied the area of the base by 2.
$6 \times 11\ cm^2 = 66\ cm^2$	Since the base of the hexagonal prism is a regular polygon, all the sides are congruent. I multiplied the area of the rectangle by 6.
$12\ cm^2 + 66\ cm^2 = 78\ cm^2$	Then I added the areas. The surface area of the hexagonal prism is about 78 cm^2.

A Checking

5. a) Use a 3-D model for a triangular prism. Trace all the faces on centimetre grid paper.

b) Count squares and part squares. About how many square centimetres is the area of each face?

c) About how many square centimetres is the surface area of the prism?

d) How could you determine the surface area without tracing all the faces?

Surface Area of a Right Prism (page 3)

B Practising

6. **a)** Trace faces of a pentagonal prism to determine the surface area.

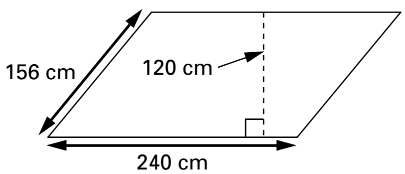
b) About how many square centimetres is the area of each tracing?

c) What is the surface area of the prism to the nearest square centimetre?

d) How did you decide which faces to trace?

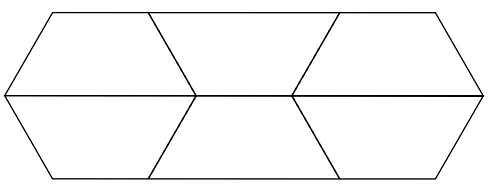
7. Stack four blue rhombus pattern blocks and repeat question 6.

8. Alex has a summer job building sandboxes at a park. This parallelogram is the base of one sandbox. The height of the sandbox is 14 cm. What area of wood, to the nearest tenth of a square metre, is needed to build the sandbox?



C Extending

9. Trace the faces of one trapezoid pattern block on centimetre grid paper. Use your tracing to determine the approximate surface area of a prism made with three layers of this design.



10. Is it true that when you double the height of a hexagonal prism, you double the surface area? Justify your answer.

11B Volume of a Right Prism (page 1)

GOAL Develop a formula to calculate the volume of a right prism, and solve volume problems.

You will need
- pattern blocks
- a calculator

Learn about the Math

Paul is designing boxes for some games. He wants the box for each game to be a right prism with a different base.

? How can Paul calculate the volume of each box?

A. Use blue rhombus pattern blocks to model the volume of a box with a rhombus base. Start with a prism made with one rhombus pattern block. How does a row in this table show the volume of your prism?

Base of prism	Height of prism	Volume of prism
Area of the base of a rhombus pattern block	1 layer	1 rhombus pattern block
Area of the base of a rhombus pattern block	2 layers	2 rhombus pattern blocks

B. Add a second rhombus pattern block to your prism. How does the next row in the table show the volume of this prism?

C. Add a third rhombus pattern block to your prism. Copy the table and record another row for your prism.

D. Continue step C by adding one rhombus pattern block at a time to your prism until the volume is 6 rhombus pattern blocks.

E. Suppose you used hexagon pattern blocks. How would the table be the same? How would it be different?

Reflecting

1. What is the relationship between the area of a rhombus and the volume of a prism with the rhombus as the base? How is this relationship the same as the relationship between the area of a rectangle and the volume of a rectangular prism?

2. How can Paul use your method to calculate the volume of each of his boxes?

3. Write a formula that describes how to calculate the volume of any right prism.

4. a) Explain why volume is expressed in cubic units, such as cm^3 or m^2, with the exponent 3.

b) Explain why the area of a base is expressed in square units, such as cm^2 or m^2, with the exponent 2.

Volume of a Right Prism (page 2)

Work with the Math

Example: Determining the capacity of a right prism

An aquarium has a trapezoid base. The parallel sides of the base are 40 cm and 30 cm. They are 20 cm apart. The aquarium is 25 cm high. What is the capacity of the aquarium in litres?

Solution

Area of a trapezoid	I calculated the area of the base of the aquarium.
$= (a + b) \times h \quad 2$	
$= (40\text{ cm} + 30\text{ cm}) \times 20\text{ cm} \quad 2$	
$= 700\text{ cm}^2$	
Volume of a right prism	I multiplied the area of the base of the aquarium by the height of the aquarium.
= area of base $\times$ height	
$= 700\text{ cm}^2 \times 25\text{ cm}$	
$= 17\,500\text{ cm}^3$	
$= 17\,500$ mL	$1\text{ cm}^3 = 1$ mL so the capacity of the aquarium is 17 500 mL.
$= (17\,500 \quad 1000)$ L	Since 1000 mL = 1 L, I divided the number of millilitres by 1000.
$= 17.5$ L	The capacity of the aquarium is 17.5 L.

A Checking

5. Express each as a volume in cubic centimetres.

a) 400 mL **b)** 2 L **c)** 16 L **d)** 7456 mL

6. Determine the volume of each right prism.

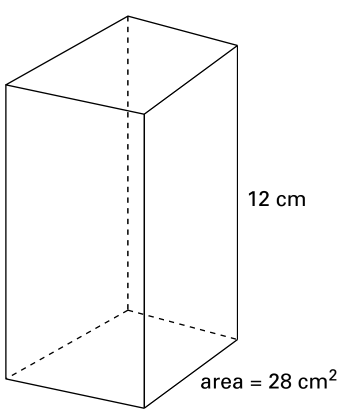
a) A right prism has a base with an area of 16 cm^2 and a height of 9 cm.

b) A right prism has a trapezoid base whose parallel sides are 5 cm and 9 cm. The parallel sides are 5 cm apart. The height of the prism is 7 cm.

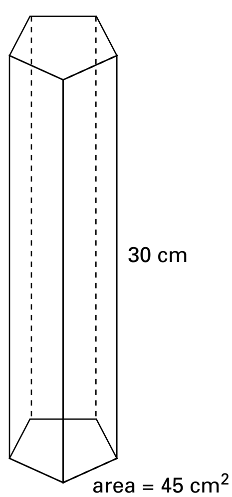
B Practising

7. Calculate the volume of each prism.

a)



b)

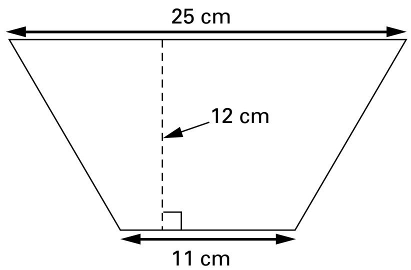


Volume of a Right Prism (page 3)

8. a) What is the capacity of each prism in question 7 in millilitres?

b) What is the capacity of each prism in question 7 in litres?

9. A box has this trapezoid base. The height of the box is 1.5 m. What is the volume of the box in cubic centimetres?



10. A carton in the shape of a right prism has the base area 8 cm^2 and a volume of 560 cm^3. What is the height of the prism?

11. A container has a capacity of 18 L and a height of 400 cm. What is the area of the base?

C Extending

12. a) Does a hexagon pattern block have the same volume as a stack of six triangle pattern blocks? Justify your answer.

b) Does a hexagon pattern block have the same surface area as a stack of six triangle pattern blocks? Justify your answer.

13. Explain how a formula for the volume of a right prism supports your answers.

a) What happens to the volume of a trapezoidal prism when the height is tripled?

b) What happens to the volume of a trapezoidal prism when the area of the base is halved?

c) Are the answers for parts (a) and (b) the same for all right prisms?